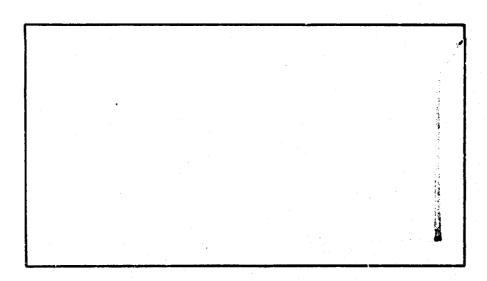
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13. ABSTRACT

The possibility of replacing large highly directive parabolic antennas for satellite tracking with a phased array is investigated. It is shown that a non-planar array with a circular horizontal cross section and an elliptical vertical cross section has a far field radiation pattern suitable for satellite tracking. The problem is formulated for solution by a high speed digital computer and an analysis of the performance of the optimal array design is presented.

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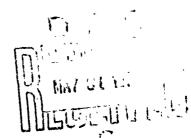
A PHASED ARRAY ANTENNA WITH HEMISPHERIC SCAN FOR SATELLITE TRACKING

THESIS

GE/EE/72-15

Thomas B. Markham Captain USAF

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A PHASED ARRAY ANTENNA WITH HEMISPHERIC SCAN FOR SATELLITE TRACKING

THESIS

of the Air Force Institute of Technology

Air University

in Partial Fulfillment of the

Requirements for the Degree of

Master of Science

by.

Thomas B. Markham
Captain USAF
Graduate Electrical Engineering

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Preface

This thesis is the result of my study of high gain directive array antennas and my prior experience as a Satellite Operations Controller with the Air Force Satellite Control Facility. The phased array seemed to be the answer to the mechanical difficulties inherent to large highly directive parabolic antennas. However, a planar array cannot be scanned to the limits required unless some means of changing its position is provided. This does not eliminate the mechanical difficulties, only alters them. Therefore, I turned my attention to a non-planar array with as much symmetry as possible to reduce the complexity of the solution. With the help and guidance of Dr. Russell W. Taylor, to whom I am greatly indebted for his encouragement and understanding, I proceeded to formulate the solution of the fair field pattern of an array with circular horizontal and elliptical vertical cross sections in a format suitable for solution by a high speed digital computer. The results of this study are presented in this paper. I would also like to thank Major John Pierce of the Air Force Institute of Technology Computer Center for his assistance in eliminating the many errors encountered in programming the solution and his patience during the many programs submitted for execution. I would also like to thank Mrs. Gloria McNally for her invaluable assistance in preparing this paper for submission.

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Abstract

The possibility of replacing large highly directive parabolic antennas for satellite tracking with a phased array is investigated. It is shown that a non-planar array with a circular horizontal cross section and an elliptical vertical cross section has a far field radiation pattern suitable for satellite tracking. The problem is formulated for solution by a high speed digital computer and an analysis of the performance of the optimal array design is presented.

A PHASED ARRAY ANTENNA WITH HEMISPHERIC SCAN FOR SATELLITE TRACKING

i. Introduction

Background

The Air Force Satellite Control Facility, (SCF), is charged with the responsibility of tracking, commanding, and gathering telemetry data from Department of Defense satellites. To accomplish this task the SCF is presently using a parabolic antenna with a diameter of sixty feet. A serious problem arises when tracking a low orbit (approximately 100 nautical miles) satellite during a near overhead pass. This problem is the extreme difficulty of rotating the large mass of the antenna as much as 180° in a matter of seconds without excessive lag or overshoot. This problem could be overcome by utilizing a fixed antenna with a steerable beam, i.e., a phased array. However, this solution poses another, equally serious, problem in that phased array technology at the present time does not include an array with acceptable beam characteristics at scan angles greater than approximately 60° from broadside. In order to accomplish the task of gathering data from a satellite it is necessary to maintain radio contact with it for as long as possible. In other words, the ground based tracking antenna must be capable of scan angles symmetric about the vertical axis for the entire range of 90° from the axis

Purpose

The purpose of this study is to investigate the feasibility of using a phased array to accomplish the SCF mission.

Approach

Since the technology to date does not include the large scan angles required, it was decided to abandon the planar phased array and investigate the

possibility of using a non-planar array. Since the requirement exists to scan 360° in azimuth, consideration was limited to geometry symmetric about the vertical axis. This indicated an array with a circular cross-section in the horizontal plane. The possible vertical cross-sections included conic sections, rectangles, trapezoids, and truncated conic sections. Since consideration of all possible cross-sections was beyond the scope of this study, the geometry was further restricted to the most general conic section, the ellipse. With the general geometry of the array fixed, the CDC 6600 digital computer was programmed to calculate the fair field radiation pattern of the specified array with the eccentricity of the ellipse, the number of elements in the array, and the phasing of the elements as input variables. The field pattern was calculated for various array designs at various scan angles in order to arrive at an optimal design.

II. Theory

Linear Array Theory

If an array is considered as a single source made up of differential radiators, then, ignoring the effects of mutual coupling, the resultant radiation pattern is a superposition of the field contributions of each elementary source. This means that, if the phase and amplitude of each element can be chosen such that the cancellation of the fields in all but the desired direction results, a highly directive antenna can be constructed. It is found that, in general, the desired results can be achieved by using identical elements equally spaced. By using this configuration, the total array pattern can be obtained from the product of the radiation pattern of the individual element and the pattern of an identical array of isotropic radiators, as will be shown in the discussion to follow (Ref 7).

Consider the simplest array of all, an array of two identical infinitesimal current elements located on the z-axis with a spacing, d, as shown in Fig. 1. The field at the point, P, far removed from the array, produced by the element located at the origin is expressed by the equation

$$E_{\theta} = \frac{i k_{o} Z_{o} I_{o} \Delta Z}{4 \pi r} \sin \theta e^{-i k_{o} r}$$
 (1)

where k_0 is the free space wave number, Z_0 is the characteristic impedance of free space, l_0 is the excitation current of the element and Δ Z is the length of the element. The field of P due to the other element can be expressed by the same equation with r replaced by R. From the law of cosines

$$\Re = (r^{2} + d^{2} - 2 r d \cos \theta)^{\frac{1}{2}}$$
 (2)

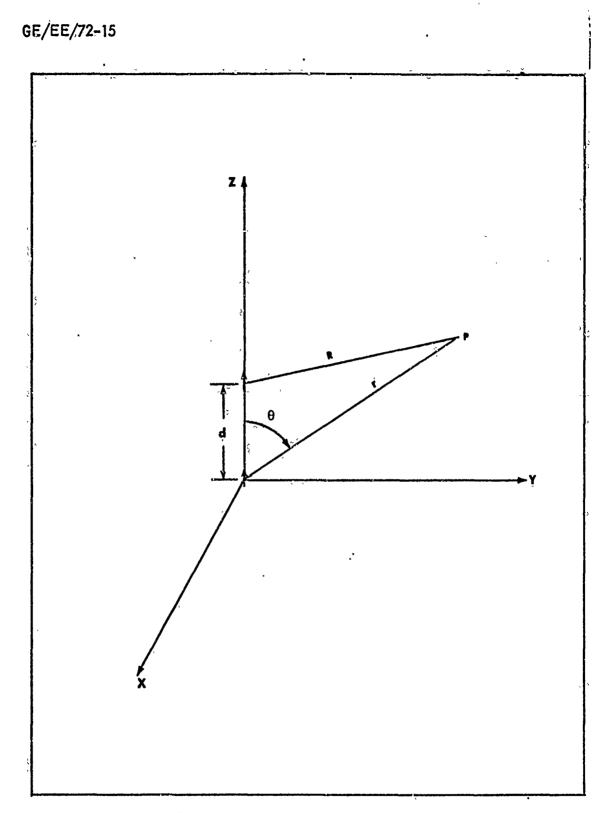


Fig. 1. Array of Two Elementary Current Elements

or, alternatively,

$$R = r \left[1 - \frac{2 d \cos \theta}{r} + \left(\frac{d}{r} \right)^2 \right]^{\frac{1}{2}}$$
 (3)

This latter equation may be expanded by means of the binomial theorem in powers of d/r. Since it is specified that $r \gg d$, the first two terms of the expansion are sufficient for phase calculations with negligible error. In addition, $R \approx r$ for far field calculations. Using these approximations the field at P due to element number 2'is expressed by

$$E_{\theta} = \frac{i k_{o} Z_{o} I_{o} \Delta Z}{4 \pi r} \sin \theta e^{-i k_{o} (r - d \cos \theta)}$$
(4)

By superposition the total field at P is given by

$$E_{\theta} = \frac{i k_{0} Z_{0} I_{0} \Delta Z}{4 \pi r} \left[e^{-i k_{0} r} + e^{-i k_{0} (r - d \cos \theta)} \right] \sin \theta$$

or

$$E_{\theta} = \frac{i k_{0} Z_{0} \delta_{0} \Delta Z}{4 \pi r} = \frac{-i k_{0} r}{e} \left[1 + e^{i k_{0} d \cos \theta} \right] \sin \theta.$$
 (5)

Figure 2 shows graphically the variation of E $_{\rm A}$ with $_{\rm A}$.

It is easily seen from the above analysis that if arbitrary identical antenna elements replace the infinitesimal current elements above, the resultant field could be written

$$E_{\theta} = f(\theta, \varphi) \frac{e}{r} \qquad \left[\begin{array}{c} -i k_{0} r \\ 1 + e \end{array} \right]$$
 (6)

where $f(\theta, \phi) \exp(-j k_0 r)/r$ represents the field due to one element located at the origin of the coordinate system.

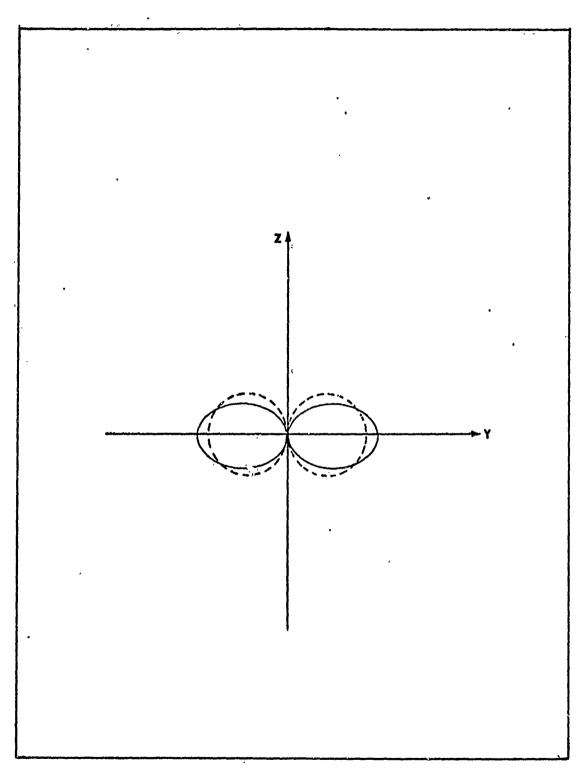


Fig. 2. Radiation Pattern of Single Dipole (----) and Array of Figure 1 (----)

Generalizing these results still further, for an array of N identical antenna elements spaced a uniform distance, d, apart along the z-axis, the far field can be expressed as

$$E_{\theta} = f(\theta, \varphi) \frac{e^{-i k_0} r}{r} \left[1 + \sum_{n=1}^{N-1} e^{i k_0 n d \cos \theta} \right]$$
 (7)

with the restriction that $r \gg Nd$ for the far field approximations to hold. This expression for the far field may be considered as the product of the element factor, $f(\theta,\phi) \exp(-ik_{_{\rm O}}r)/r$, and an array factor, A, with

$$A = 1 + \sum_{n=1}^{N-1} e^{\int_{0}^{\infty} k_{n} d \cos \theta}$$

If the amplitude and phase of the individual elements are varied such that the nath element has amplitude, C_n , and phase, $\exp(i\alpha_n)$, referenced to the element at the origin, the array factor, A, becomes

$$A = 1 + \sum_{n=1}^{N-1} c_n e^{\int_0^1 k_0 n d \cos \theta} e^{\int_0^1 \alpha_n}$$
(8)

Now consider the linear array of Fig. 3 with a uniform phase progression denoted by

$$\alpha_n = -n \, d \, k_0 \cos \, \theta_0 \qquad \qquad (9)$$

where θ_0 is a constant denoting the desired direction of the main beam. Set $C_n = 1$ for all n. Then the magnitude of A is given by

$$|A| = \begin{vmatrix} i k_0 d(\cos \theta - \cos \theta_0) & i k_0 2d(\cos \theta - \cos \theta_0) \\ + e & + e & + \end{vmatrix}$$

$$i k_0 (N-1) d(\cos \theta - \cos \theta_0)$$

$$... + e \qquad (10)$$

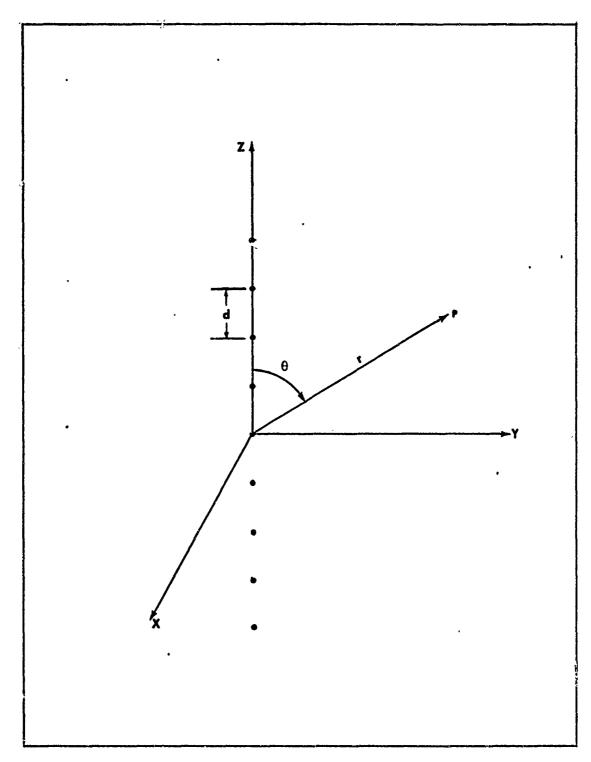


Fig. 3. Linear Array of Elementary Radiators

Equation (10) is a geometric progression with a ratio $\exp[ik_0 r(\cos \theta - \cos \theta_0)]$ which can be expressed as

IAI =
$$\frac{\begin{vmatrix} N k_0 d (\cos \theta - \cos \theta_0) \\ \frac{k_0 d (\cos \theta - \cos \theta_0)}{2} \end{vmatrix}}{\sin \frac{0}{2}}$$
 (11)

Let $x = k_0$ d (cos $\theta = \cos \theta_0$). Then it is evident that as x approaches zero, A approaches N. Nulls in the pattern occur when IAI = 0, or when $N \times /2 = \pi$, 2π , 3π , ..., $(N-1)\pi$. Since the maximum value of IAI occurs at x = 0, the main beam is indeed in the direction $\theta = \theta_0$. This fact indicates that the direction of the main beam may be shifted by altering the phase shift of the elements, α_n . It is now of interest to examine the field pattern as the direction of the main beam is shifted through large angles. (Refer to Fig. 4.) Designating the position of the first null to the left of the main beam as θ^+ and the first null to the right as θ^- it is seen that

$$k_o d \left[\cos \left(\theta_o + \theta^+\right) - \cos \theta_o\right] = \frac{2\pi}{N}$$
 (12)

and

$$k_o d \left[\cos \left(\theta_o - \theta^{-}\right) - \cos \theta_o\right] = \frac{-2\pi}{N}. \tag{13}$$

Assuming θ^+ and θ^- to be very small (as indeed will be shown to be true for large N) the approximations $\cos \theta = (1 - \theta^2/2)$ and $\sin \theta = \theta$ may be used. With these substitutions, equations (12) and (13) become

$$k_0 d \left[-\theta^+ \sin \theta_0 - \frac{(\theta^+)^2}{2!} \cos \theta_0 \right] = \frac{2\pi}{N}$$
 (14)

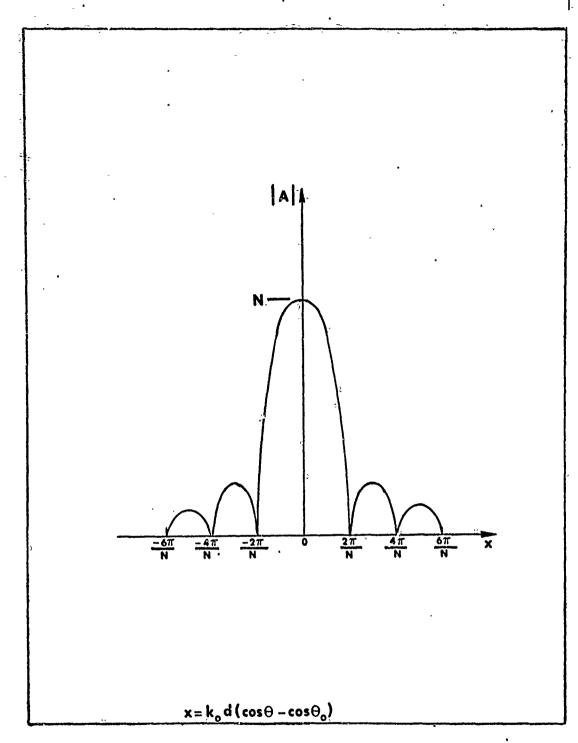


Fig. 4. Array Factor of Array in Fig. 3

and

$$k_{o} d \left[\theta^{-1} \sin \theta_{o} - \frac{\left(\theta^{-1}\right)^{2}}{2} \cos \theta_{o}\right] = \frac{-2\pi}{N}. \tag{15}$$

Assuming $\theta_0 > 9^{\frac{1}{2}}$, the quadratic terms in equations (14) and (15) can be dropped with the resulting expressions for θ^+ and θ^- .

$$\theta^{\dagger} = \theta^{-} = \frac{2\pi}{N \, k_{o} \, d \sin \theta_{o}} = \frac{\lambda_{o}}{N \, d \sin \theta_{o}}. \tag{16}$$

Thus it is seen that if Nd/λ_0 is large and θ_0 sufficiently large, θ^+ and θ^- are indeed small as was assumed earlier. It is also evident that as θ_0 varies from $\pi/2$, θ^+ and θ^- increase until the assumptions made to obtain equation (16) are no longer valid, i.e., the quadratic terms in equations (14) and (15) can no longer be dropped. For $\theta_0=0$, it is seen that

$$\theta^{\pm} = \theta^{-} = (2\lambda_{o}/Nd)^{\frac{1}{2}}. \tag{17}$$

This condition results in an endfire array. While a linear array can produce a narrow beam in the 9-direction, Fig. 5 shows that the beam is in fact symmetric about the endfire axis. In addition, the broadening of the beam as 9 o approaches zero renders it unsuitable for large scan angles.

Planar Arrays

The above theory of linear arrays is readily expanded to the case of the two-dimensional array of equally spaced arrays of equally spaced elements (see Fig. 6). This array may be considered to be a linear array along the y-axis whose elements are linear arrays oriented in the z-direction. From the above analysis it is seen that the far field of this array is the product of an array factor, A_{ν} , and an element factor, A_{h} , where the element factor is the product of the individual element factor and the array factor of a linear array is the z-direction.

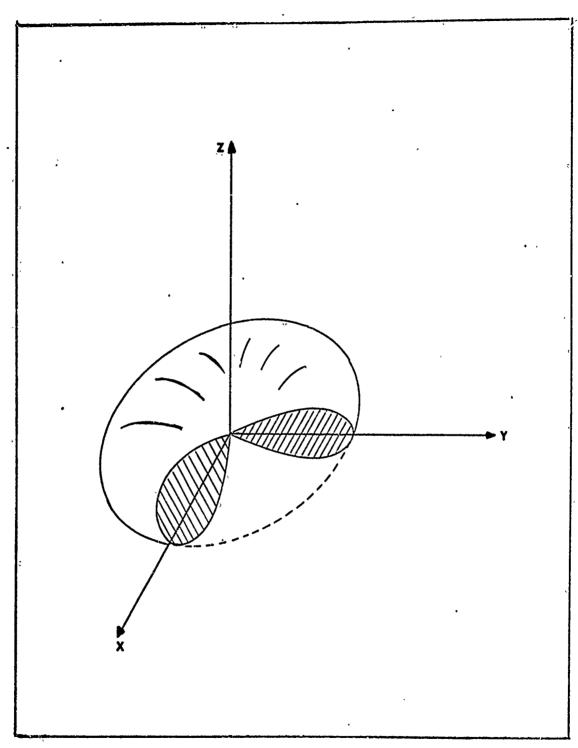


Fig. 5. Radiation Pattern of Linear Array of Fig. 3

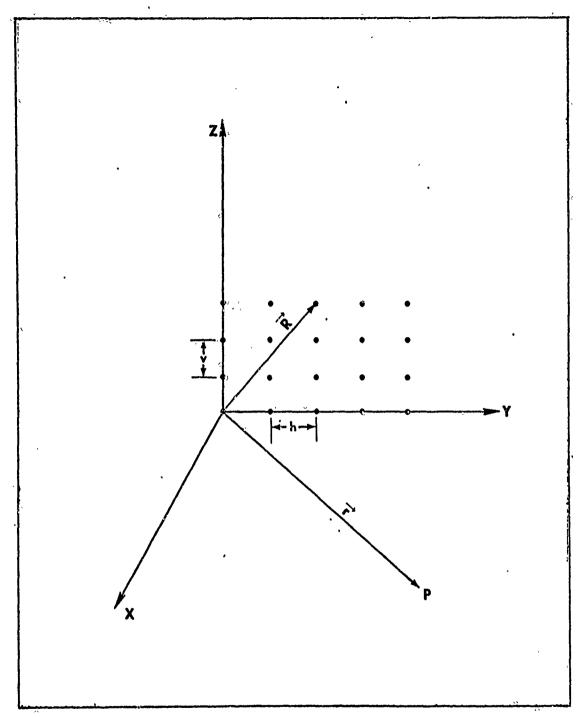


Fig. 6. Two Dimensional Linear Array

$$E_{\theta} = f(\theta, \varphi) \frac{e^{-\theta k_0} r}{r} \left[1 + \sum_{m=1}^{M-1} e^{\int m(k_0 v \cos \theta - \theta)} \right]$$

$$\left[1 + \sum_{n=1}^{N-1} e^{i n (k_0 h \sin \theta \sin \phi - \alpha)}\right]$$
(18)

where α and β are the progressive phase shifts in the horizontal and vertical directions respectively. Alternatively this can be written

$$E_{\theta} = f(\theta, \varphi) \frac{e^{-jk_0 r}}{r} \sum_{m=0}^{M-1} e^{jm(k_0 y \cos \theta - \beta)} \sum_{n=0}^{N-1} e^{jn(k_0 h \sin \theta \sin \varphi - \alpha)}$$

$$= f(r, \theta, \varphi) A_h A_v$$
(19)

where A_{ν} is the array factor calculated above in the analysis of the linear array of elements along the z-axis, and A_{h} is the array factor of an identical array oriented along the x-axis. Figure 7 shows that the resulting field from the two-dimensional array is no longer symmetric about the endfire axis (if in fact an andfire axis could be defined) but is a pencil beam on either side of the array.

Volume Arrays

Logic suggests that an extension of the theory of linear arrays to the case of a three-dimensional array made up of a linear array of twodimensional arrays described above should yield a single pencil beam which could be scanned in any direction desired. Indeed, extending linear array theory to the volume array of Fig. 8 yields an expression for the far field

$$E_{\theta} = f(r, \theta, \varphi) A_{x} A_{y} A_{z}$$
 (20)

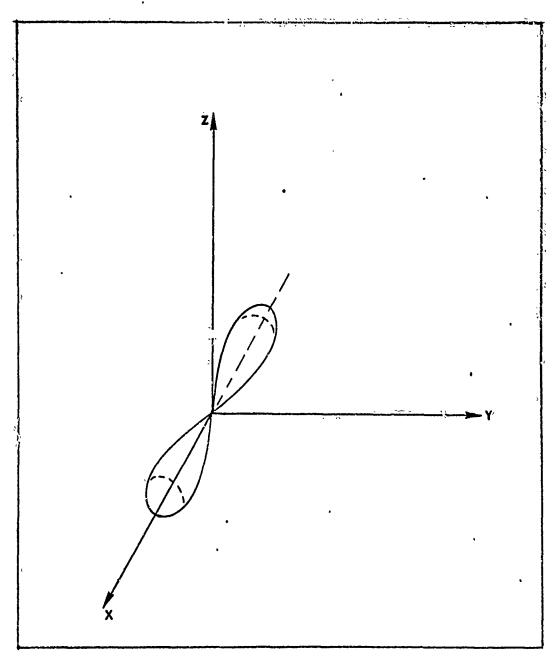


Fig. 7. Radiation Pattern of Array of Fig. 6

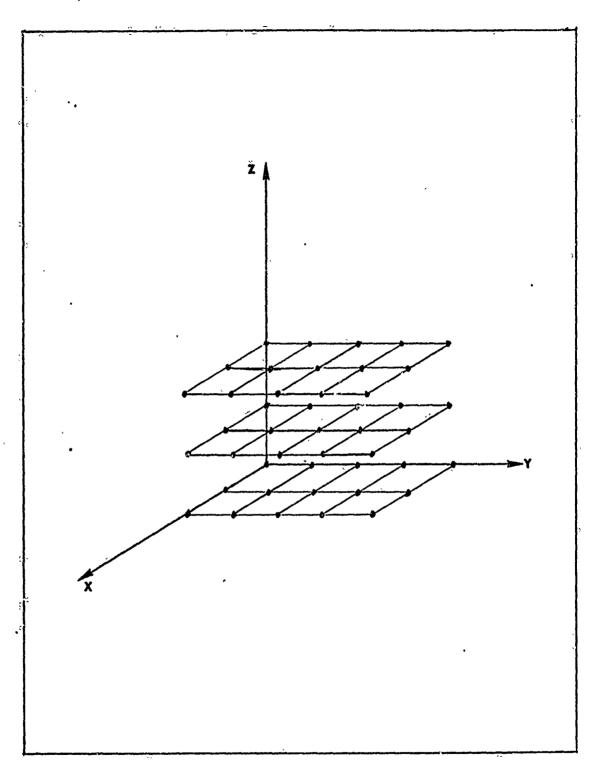


Fig. 8. Volume Array of Elementary Radiators

where

$$\dot{A}_{x} = \sum_{m=0}^{M} e^{i m (k_{0} a \sin \theta \cos \phi - \alpha)}$$
(21)

$$A_{y} = \sum_{n=0}^{N} e^{i n (k_{0} b \sin \theta \sin \phi - \beta)}$$
(22)

and

$$A_{z} = \sum_{\dot{p}=0}^{p} e^{i\dot{p}\cdot(k_{o}\cos\theta - \zeta)}$$
(23)

with α , β , and ζ the progressive phase shifts in the x, y, and z directions respectively. It is easily seen that an analytic solution to equation (20) would be extremely difficult for a large array. However, this equation lends itself readily to solution by a high speed digital computer.

Mutual Coupling

It has been shown that the radiation pattern and input impedance of individual elements in an array are changed by the effects of mutual coupling.

It is also known that these effects become more pronounced at large scan angles (Ref 9). Cheng states:

"To the author's knowledge, no work has been published on array synthesis or optimization for mutually coupled aperture-type radiators." (Ref 1:1671)

For this reason effects of mutual coupling were not considered in this study.

18. A Non-planar Array for Satellite Tracking

Design Considerations

The task of tracking an orbiting satellite dictates certain restrictions on the geometry of the array to be used. Complete symmetry about the vertical axis specifies the optimum horizontal cross-section as a circle. The requirement to track from horizon to horizon precludes the use of a planar array due to the unsatisfactory beam characteristics at large scan angles. For these reasons the geometry was chosen to be a surface described by the equation

$$x^2 + y^2 + z^2 / E = R^2$$
 (24)

It can be seen from the analysis of the volume array that the beamwidth is a function of the number of elements in the array, the spacing of the elements, the wavelength of the radiated wave, and the scan angles θ and ϕ . Due to the symmetry chosen, the ϕ -dependence of the beamwidth was eliminated. Also the operating frequency (1.7 to 2.3 GHz) fixed the wavelength.

The operating frequency also suggested the use of rectangular aperatures as the basic radiating element. Design consideration was limited to a rectangular aperture operating in the TE_{10} mode. This restriction dictated an aperture 0.8 wavelength wide and 0.4 wavelength high. This further indicated a spacing of the elements of one wavelength in the φ -direction and 0.5 wavelength in the θ -direction. The radius at the base of the antenna depends on the number of elements and the spacing. These restrictions fixed all variables with the exception of the eccentricity of the vertical cross-section and the number of elements at the base of the array. Figure 9 shows the basic geometry considered in this study.

Field Calculations

With the geometry of the antenna specified, the field of the antenna can be calculated from the equation for the volume array

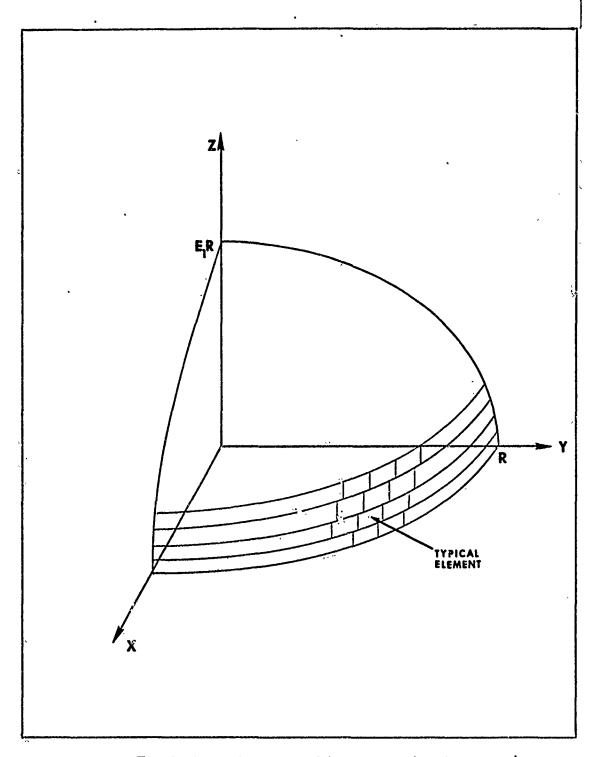


Fig. 9. Basic Geometry of Array Considered

$$E_{\theta} = \sum_{m=0}^{M} \sum_{n=0}^{N} \sum_{p=0}^{P} f_{mnp} (r, \theta, \varphi) C_{mnp} A_{x, y, z}$$
(25)

where f_{mnp} (r,θ,ϕ) is a function of position of the mnp-th element in the array and C_{mnp} = 0 for all elements not on the surface described by equation (24). Equation (25) is now rewritten in polar form as

$$E_{\theta} = \frac{e^{-ik_0 r}}{r} \sum_{\theta=0}^{\pi/2} \sum_{\varphi=0}^{2\pi} f(\theta, \varphi) e^{-ik_0 f_1(\theta, \varphi)} e^{ik_0 f_2(\theta, \varphi)}$$
(26)

where f, f_1 , and f_2 are functions of the beam coordinates, θ_0 and ϕ_0 , the coordinates of the field point, θ and ϕ , and the location of the element in the array. (See Appendix A for derivation of f, f_1 , and f_2 .)

IV. Computer Simulation

The design of the array described in Chapter III was accomplished using six computer programs written for the CDC 6600 computer using the Fortran Extended compiler. (See Appendix B for program listings.)

Program Pattern

Program Pattern is a program which computes the field contribution of each element in the array at the specified field point, sums the individual contributions and prints the value in tabular form. Field calculations are performed in a specified φ -plane for one degree increments of θ from $\theta=0^\circ$ to $\theta=90^\circ$. The program, given the number of elements around the base of the array and their spacing and the ratio of the height of the array to the radius of its base, will also calculate and store for later use the essential geometry for the field calculations (i.e., the number of elements in each circular subarray, the arc length between adjacent subarrays, total arc length of the array from base to top, and total number of elements in the array). The program calculates the field by solving equation (26) using as input variables the number of elements in the base of the array, the horizontal spacing of the array elements, the φ -coordinate of the field point, the angular coordinates of the beam, the ratio of the height of the array to the radius of the base, and the value of θ at which it is desired to begin calculations. This last input was provided in the event that only a particular region of space is of interest. Output from the program consists of a listing of the geometry of the array, a tabular listing of θ , E_{θ} , $IE_{\theta}I$, and the phase of E_{θ} , and the θ coordinate and magnitude of the maximum value of IE $_{\theta}$ I. In addition, the listing of θ and \boldsymbol{E}_{θ} is reproduced on punched cards for use with a plotting routine for graphical display of the array pattern as a function of θ .

Program Two

Program Two is a modification of Program Pattern which calculates the field for 0.1° increments of θ starting at the center of the main beam. This

program was provided to examine the field pattern in the immediate vicinity of the main beam in more detail.

Programs Three and Four

Programs Three and Four are modifications of Programs Pattern and Two respectively which compute the variation in the field pattern in the ϕ -direction. While the first two programs hold ϕ constant while θ is varied, the latter two hold θ constant while varying ϕ .

Program Beam

The fifth program in the series is still another modification of Program Pattern which calculates the magnitude and phase of the field at the center of the main beam as it is scanned from $\theta = 0^{\circ}$ to $\theta = 90^{\circ}$ in one degree steps. This program, like the first four, provides the output data in punched card format for use in the standard plot subroutines available with the CDC 6600.

Program Optimum

The sixth and last program is an optimization routine designed to find the ratio of the height to base radius of the array which yields the most nearly uniform gain as the beam is scanned from $\theta=0^\circ$ to $\theta=90^\circ$. Let H/R be the ratio of height to radius of the base and Ξ_2/Ξ_1 the ratio of the magnitudes of the beam at $\theta=90^\circ$ and $\theta=0^\circ$. The program takes an initial value of H/R and calculates Ξ_2/Ξ_1 for this configuration. H/R is then increased in steps of 0.1 until the ratio Ξ_2/Ξ_1 decreases. H/R is then decreased in steps of 0.01 until the ratio again begins to decrease. This procedure is continued until the optimum value of H/R is obtained. This ratio is then printed along with the geometry of the array. The design resulting from this program is then analyzed using the first five programs described above.

V. Results, Conclusions, and Recommendations

Results

An analysis of the output of Program Pattern, executed for an hemispherical array revealed that an array with the basic geometry described in Chapter III would indeed yield improved performance over a planar array at large scan angles. Figure 10 is a computerized plot of the magnitude of the Effield versus scan angle for an hemispherical array. It should be noted that this array pattern does not have the sharp drop in field intensity as the beam is scanned past 60° which is found in the planar array. With the fact established that an array with this shape could be made to scan through the range required, Program Optimum was executed and an optimum ratio of height of the array to the radius of its base was computed to be 2.49. It was also found during pre-liminary runs of Program Pattern that an array with 128 elements in the base ring would yield a beamwidth less than two degrees. These results fixed the geometry of the array entirely. Figure 11 is a scale drawing of the array design which is analyzed in some detail in the paragraphs to follow.

The execution of Program Beam revealed that the gain of the array is linear within 2 db as the beam is scanned from the vertical to the horizon (see Figs. 12 and 13). The minimum gain of the array referenced to a single radiating aperture oriented normal to the direction of the beam is approximately 29 db at $\theta = 90^{\circ}$.

Analysis of the results of Program Two revealed that the beamwidth in the θ -direction is not only less than 2° throughout the range of scan but actually decreases as the scan angle increases to a value of approximately 80° . Figures 14 and 15 show the variation in beamwidth in the two directions with scan angle. The increase in beamwidth at scan angles greater than 80° can be attributed to edge effects as the beam approaches the lower edge of the array. It should be noted, however, that the maximum beamwidth of approximately 1.5°

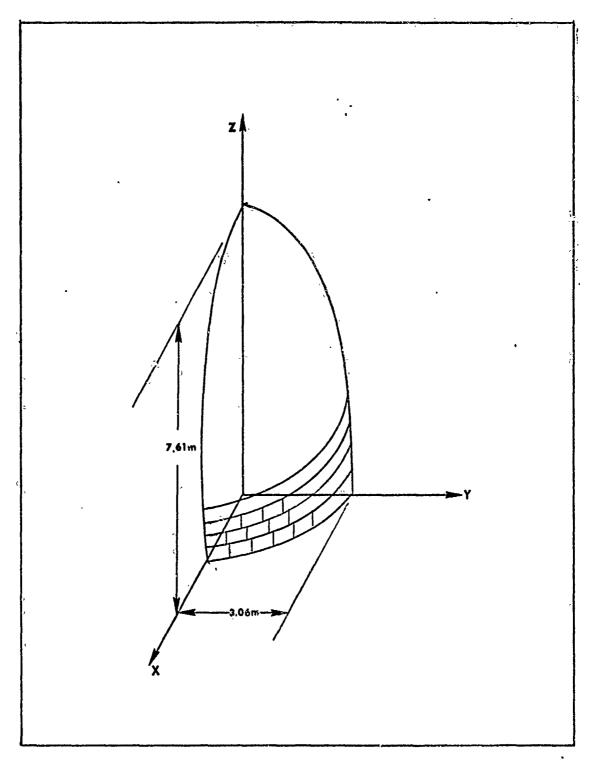


Fig. 10. Beehive Array Resulting from Execution of Program
Optimum (See Appendix C for Complete Specifications)

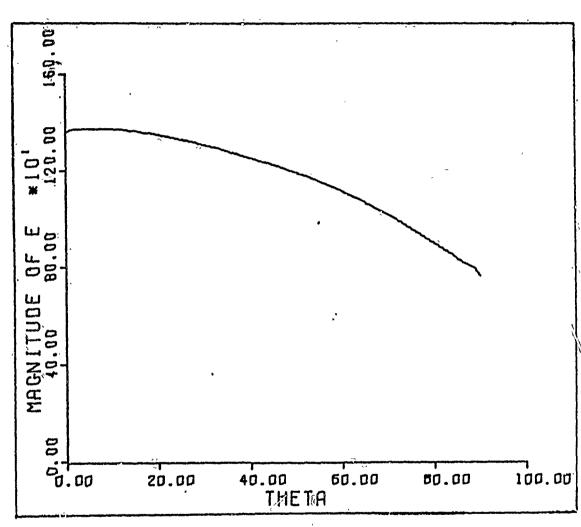


Fig. 11. $IE_{\theta}I$ vs θ for Hemispheric Array

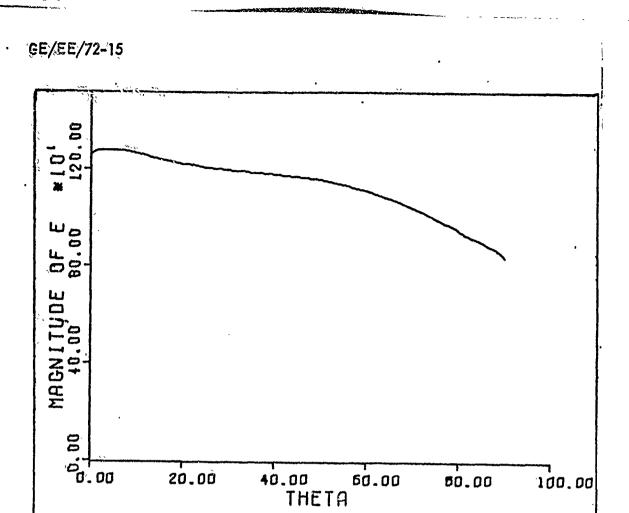


Fig. 12. Field Intensity vs Scan Angle for Optimal Array

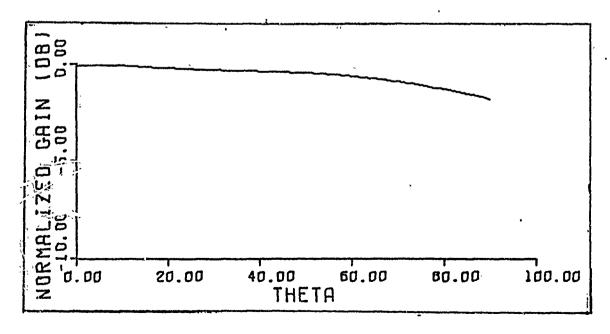


Fig. 13. Gain vs Scan Angle of Optimum Array Design

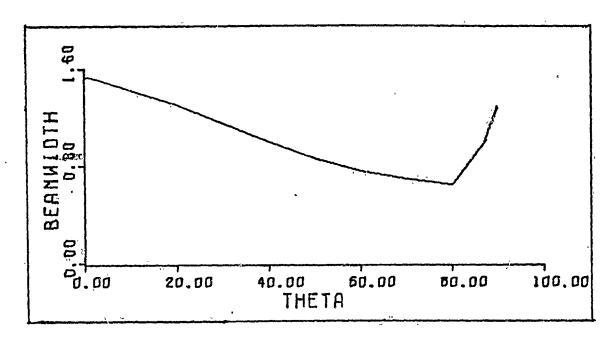


Fig. 14. 0 Beamwidth vs Scan-Angle

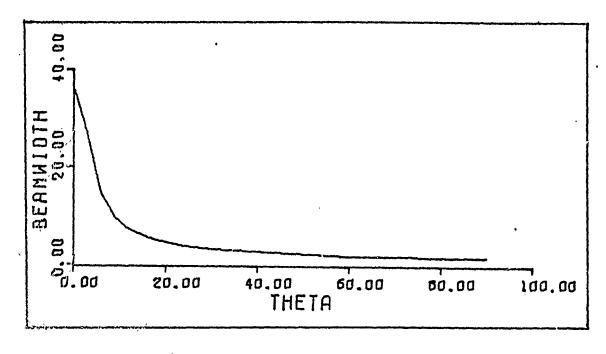


Fig. 15. φ Beamwidth vs Scan Angle

occurs not at the horizon but at a scan angle of 0° . Program Four was executed for various scan angles to determine the beamwidth and sidelobe levels in the ϕ -direction. Study was concentrated mainly in the region near the pole of the array since the symmetry of the array indicated little change in ϕ characteristics of the beam with scan angle. It was noted that the beamwidth near the pole is very broad. However, this result is inconsequential since any value for a beamwidth in the ϕ -direction near $\theta=0^{\circ}$ is meaningless.

Investigation of the field pattern on the opposite side of the array from the main beam revealed the presence of two small lobes located 136° to either side of the beam when the beam is scanned to $\theta = 60^{\circ}$. (See Fig. 16.) It is reasonable to assume that these lobes are present at other scan angles also. Further investigation of the field pattern in this vicinity revealed that the magnitude of this lobe is less than 1 percent of the magnitude of the main beam and should be of little significance.

Results of the execution of Programs Pattern, Two, Three, and Four for various beam positions revealed that the maximum sidelobe level of the array is -9 db, which is somewhat higher than the sidelobe level of a planar array with uniform spacing and excitation. However, sidelobe levels decreased as the scan angle is increased as shown in Figs. 17 through 23.

It was also discovered during analysis c the output of Program Two that the calculated beam position varies from the specified position by approximately 0.4° in the θ -direction at scan angles greater than approximately 80° . However, this should be of no significant consequence since an offset can be added to the desired scan angle to compensate for this inaccuracy.

Conclusions

It has been stated in this study that a planar array is unsuited to the task of satellite tracking due to the undesirable characteristics of the array at large scan angles. It is shown that the task can be accomplished using a non-planar array of 3796 rectangular opertures arranged in the shape of a beehive with a height equal to 2.49 times the radius of its base. The resulting phased

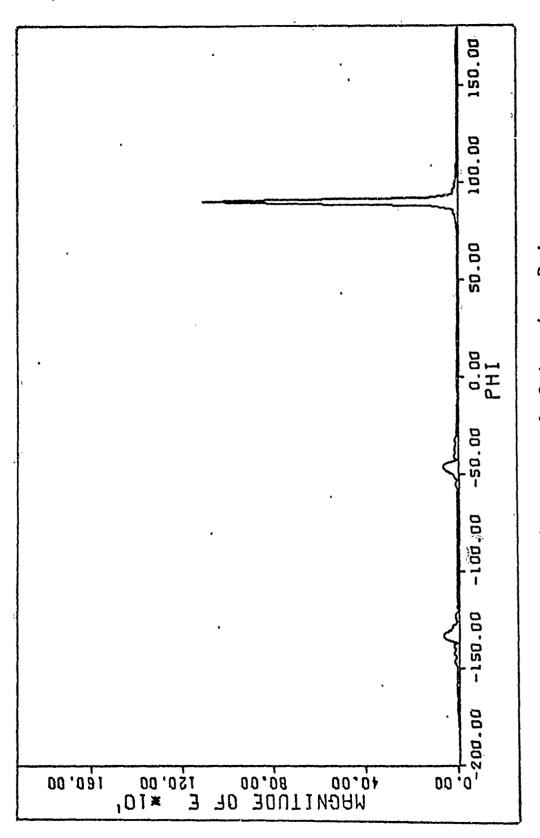


Fig. 16. Field Intensity vs & for Optimum Array Design

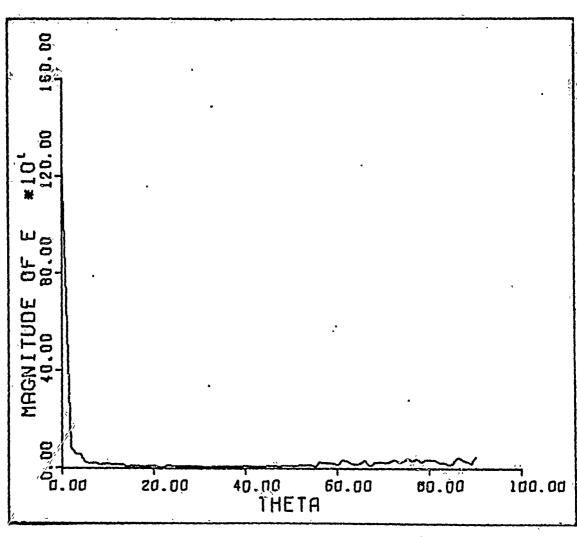
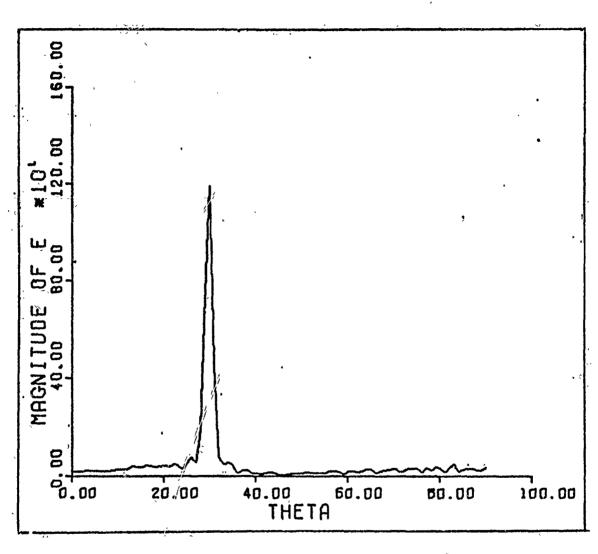


Fig. 17. Field Intensity vs θ Beam at 0^{C}



/Fig. 18. Field Intensity vs θ . Beam at 30°

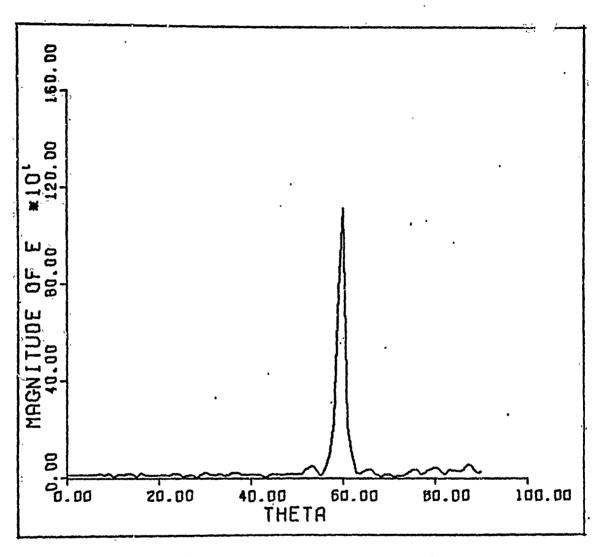


Fig. 19. Field Intensity vs θ . Beam at 60°

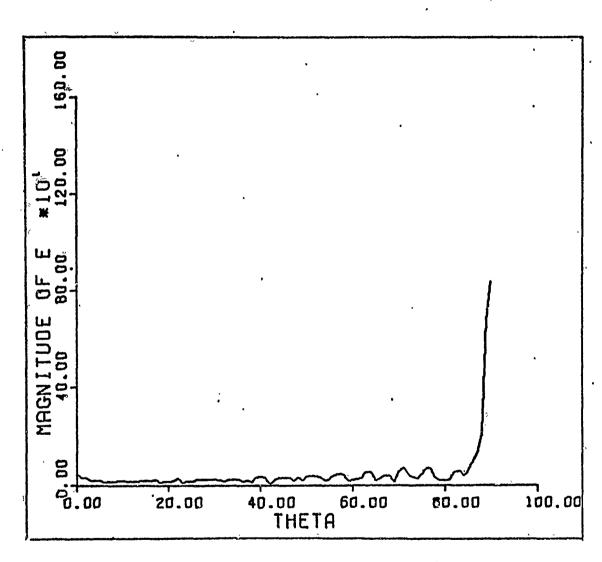


Fig. 20. Field Intensity vs 0. Beam at 90°

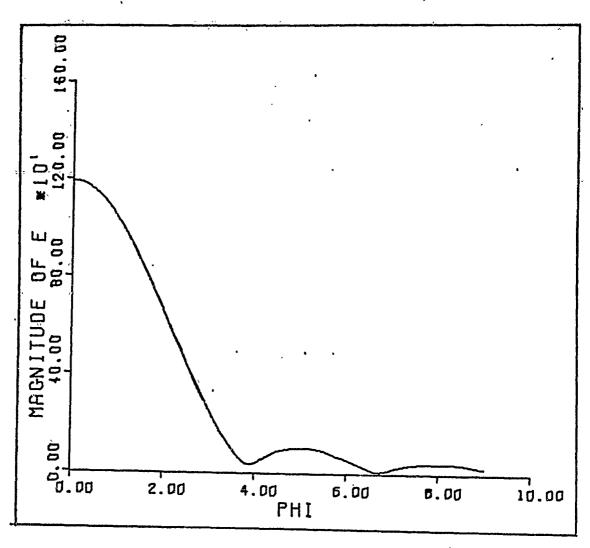


Fig. 21. Field lintensity vs ϕ . Beam at 30°

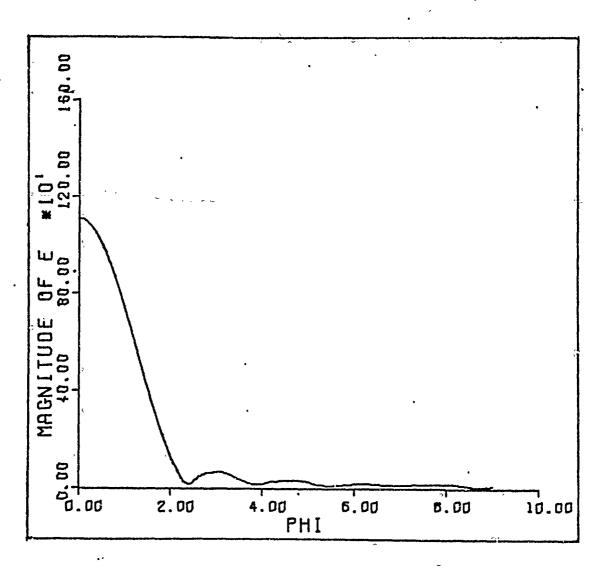


Fig. 22. Field Intensity vs φ . Beam at 60°

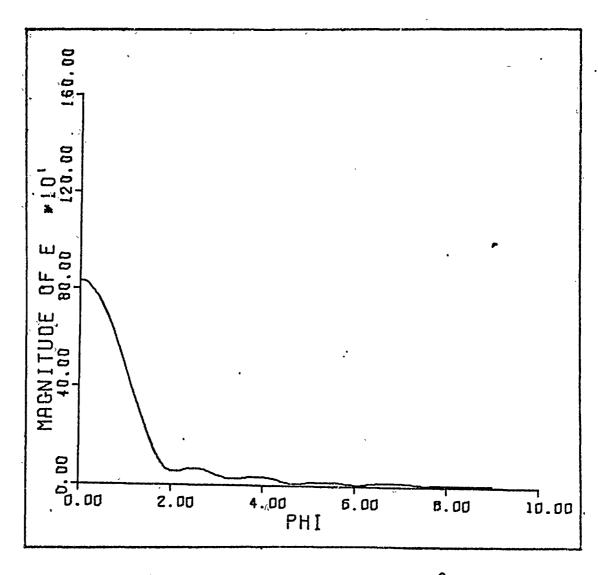


Fig. 23. Field Intensity vs φ . Beam at 90°

The gain is very nearly uniform throughout the range of scan required. While the maximum sidelobe level has deteriorated from that of the planar array, there are techniques such as non-uniform excitation and spacing available to reduce the sidelobe levels to acceptable values.

Recommendations

It is recommended that further studies of the beehive array be conducted to determine the effects of mutual coupling and the possibilities of using the technique of iris loading of the apertures to minimize these effects (Ref 6). The effect of non-uniform spacing and excitation of the elements in the array could be studied to determine whether performance of the array is improved (Ref 4).

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Appendix A

Derivation of Equations For Field Calculations

The far field of an elementary antenna element at any point in space may be expressed as

$$E_{\theta} = f(\theta, \varphi) \frac{e^{-j k_0 d}}{d} e^{j \alpha}$$
 (27)

where d is the distance from the element to the field point, $f(\theta,\phi)$ is determined by the orientation and excitation of the element, and α is the phase relative to some chosen reference.

For the geometry specified in Chapter III,

$$d = [X^{2} + R^{2} + Z_{1}^{2} - 2 XR \sin \theta \cos (\phi - \phi_{1}) - 2RZ \cos \theta]^{\frac{1}{2}}$$
(28)

where X, R, Z₁, θ , ϕ , and ϕ ₁ are as shown in Fig. 24.

Reference Fig. 25. If the arc length along the ellipse described by the equation

$$R^2 = X^2 + Z^2/E \tag{29}$$

is very small, it may be approximated by a straight line chord for the purpose of calculating the value of X. By the Pythagorean Theorem

$$Z_1^2 + (R - X_1)^2 = (\lambda/2)^2$$
 (30)

Substitution for Z from Equation (29) yields

$$X = \frac{2R - [\lambda^{2} (1 - E) + 4E^{2}R^{2}]^{\frac{1}{2}}}{2 (1 - E)}$$
(31)

$$Z = [(R^2 - \chi^2) E]^{\frac{1}{2}}$$
 (32)

and

$$\theta_1 = \tan^{-1} (Z/X). \tag{33}$$

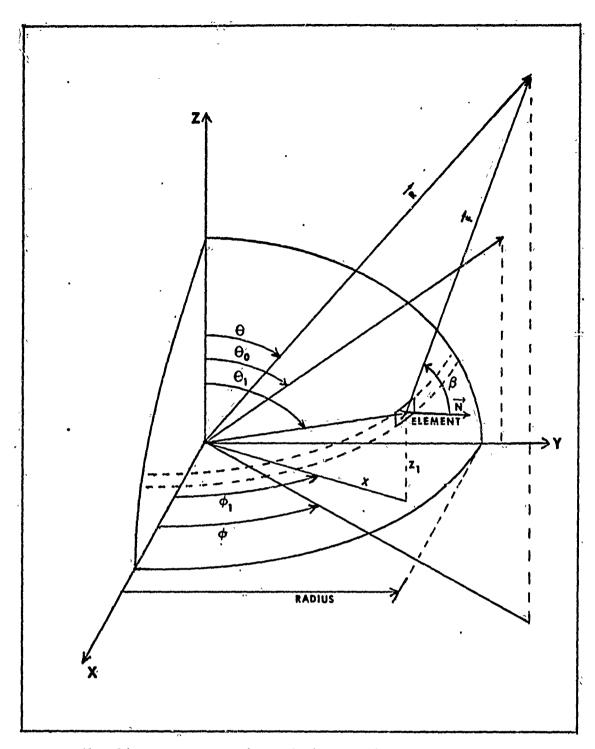


Fig. 24. Geometry Used to Calculate Field Pattern of Array

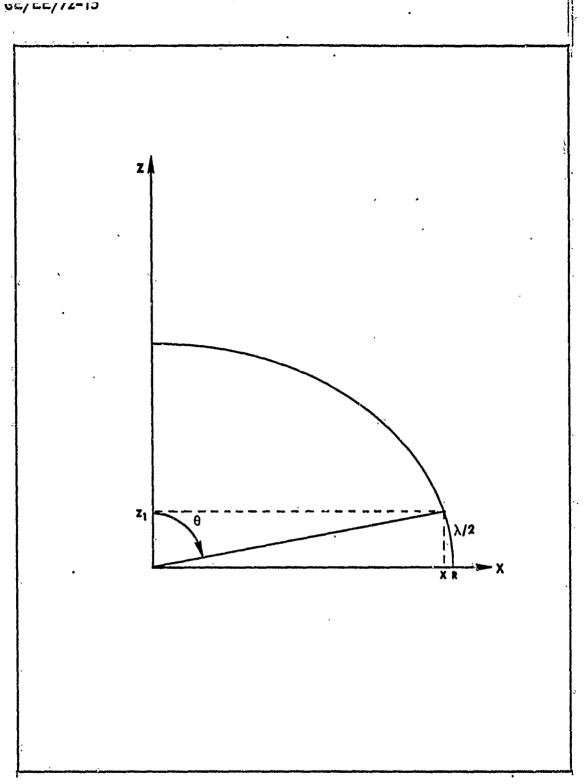


Fig. 25. Geometry Used to Calculate Radius of Second Circle of Elements

For each succeeding circle-around the array, the radius, X, (reference Fig. 26) may be calculated according to the equation

$$X_1 = X - (\lambda/2)\cos\alpha, \tag{34}$$

where

$$\alpha = \pi - \tan^{-1} \left(\frac{dz}{dx} \right)^{-1}$$

$$= \pi - 1 \left[\frac{-X_0}{[(R^2 - X^2)E]^{\frac{1}{2}}} \right]. \tag{35}$$

Arc length along the ellipse described by Equation (29) is given by

$$S = \int_{x}^{x_{1}} \left[1 + (dy/dx)^{2} \right]^{\frac{1}{2}} dx$$

$$= \int_{x}^{x_{2}} \frac{R^{2} - X^{2} (1 - E)}{R^{2} - X^{2}} dx.$$
(36)

Reference Fig. 24

$$\beta = \cos^{-1} \left[\frac{\overline{F} \cdot \overline{N}}{|F| |N|} \right] \tag{37}$$

$$\overline{F} = \overline{R}' + \overline{P}$$

$$\overline{R} = R \left[\overline{a}_{x} \sin \theta \cos \varphi + \overline{a}_{y} \sin \theta \sin \varphi + \overline{a}_{z} \cos \theta \right]$$
 (38)

$$\overline{P} = X[\overline{a}_{x} \cos \varphi_{1} + \overline{a}_{y} \sin \varphi_{1}] + \overline{a}_{z} Z_{1}$$
(39)

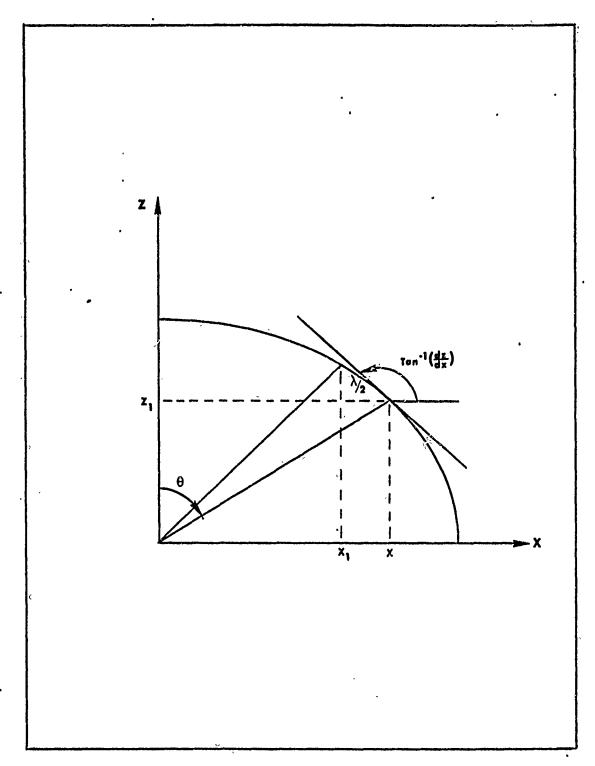


Fig. 26. Geometry Used to Calculate Third and Subsequent Circles

Appendix B <u>Computer Programs</u>

OF ARRAY * IN ARRAY ###### CIRCLE *9F3.09/) FORMAT(11919*SPACING *, I3,*/16 HAVELENGTH*,4X,*RADIUS*,F6.2,* CS*,4½,F4.1,3,* GHZ*,4X,*PHI=*,6.3,* DEGREES*,4X,*HEIGHT OF ARR F(X)=SQRT((RADIUS**2.*X**2.+E*X**2.)/(RADIUS**2.-X**2.)) DIMENSION A(91), AMAG(91), APHASE(91), NP (500), ARC(500) **BEAM COORDINATES -- THETA = *, F3.0, 2X, *PHI= B=INCREMENTAL POSITION OF FIELD POINT (IN PI/N STEPS) FORMATITHO, *MAXIMUM *, F15.5, * AT *, F3.0, *DEGREËS*, 1) ARRAY -- *, 18) THETAD AND PHIP ARE 3FAM COORDINATES IN DEGREES E1=RATIO OF HEIGHT OF ARRAY TO RADIUS OF BASE THETA=START ANGLE FOR CALCULATIONS N=NUMBER OF FLEMENTS AROUND BASE OF ARRAY FORMATCIHO, *TOTAL NUMBER OF ELEMENTS IN DEFINES CONSTANTS TO BE USED IN PROGRAM *##### NUMBER OF ELEMENTS OF ELEMENTS*) FORMAT (1H , F6.1, 3F15.5, F6.0) FORMAT (215,3,55,0, F5,3, F3,0) FORMAT (F5.3,3F15.5,2F3.0) K=2.*PI*FRE0/(3.*10.**8.) FORMAT (F6. 1, 3F15.5, F6.0) LAMBDA=3.*16.**8./FREG ,16,9X,15) GFREQ=FREQ/(10.**9.) BEGIN CALCULATIONS CUMBER*, 4X, *NUMBER BASE*) RAD=360./(2.*PI) FREQ=2.*10.**9. PI=4.*ATAN (1.0) REAL LAMBDA,K C, F5.3, * OF FORMAT (1H FORMAT (1H FORMAT (1H PSI=0. 830 700 805 803 834 808 807 2000 2000 5000 သသသ 2222 2222 CCCC သည်သ ၁၁၁၁ 2222

Z

PATTERN (INPUT, OUTPUT, PUNCH)

COMPLEX RANGE, PHASE, A, W, Z

PROGRAM

```
CALCULATES REMAINING CONSTANTS DEPENDENT ON INPUT VARIABLES
                                                                                                                                                                                                                                                                                                                                                       CF ARRAY TO TOP
          READ 100,N,L,B,THETAÖ,PHIP,E1,THETA
IF(N)503,503,502
                                                - INITIALIZES ARRAYS
                                                                                                                                                                                                                                                                                                                                                       CALCULATES ARC LENGTH FROM BASE
                                                                                                                                                                                                                                                                                                                                                                                                                                800, L, RADIUS, GFREÖ, FH, E1
803, THETAO, PHIP
808, E1, N, THETAO, PH
                                                                                                                                                                                                                                                                                        RADIUS=N*LAMBDA*L/(32.*PI)
                                                                                                                                                      AMAG(I)=APHASE(I)=D.
                                                                                                                                                                                                                                                                            PHI1=PI/2.-PHIP/RAD
                                                                                                                                                                                                                                               PHIO=PI*B/N+PI/2.
                                                                                                                                                                                                                                                                                                                                                                                                          TARC=TARC+H*F(X)
                                                                                                                                                                                                                                                                                                                                                                                             DO 134 I=1,1000
                                                                         SUMARC=0.
00 701 I=1,500
                                                                                                                                                                                                                                                                                                   H=RADIUS/1000.
                                                                                                                           00 702 I=1,97
                                                                                                                                                                                                                                                                                                                              R=260. *RADIUS
                                                                                                                                          A(I) = (0.90.)
                                                 THROUGH 702
                                                                                                                                                                                                                                                                PH=PHIO*RAD
                                                                                                  ARC(I)=0.
                                                                                                                                                                                                                                  NP(1)=N
                                                                                                                NP (I)=0
                                                                                                                                                                                                        E=E1**2
                                                                                                                                                                                                                                                                                                                                                                                  TARC=0.
                                                                                                                                                                                                                                                                                                                  X=H/2.
                                                                                                                                                                                                                                                                                                                                                                                                                                                            PUNCH
                                                                                                                                                                                                                                                                                                                                                                                                                       H+X=X
                                                                                                                                                                                                                                                                                                                                                                                                                                    PPINT
                                                                                                                                                                                                                                                                                                                                                                                                                                               PPINT
                                                                                                                                                                                                                      M1=1
            202
                                                                          505
                                                                                                                                                      702
                                                                                                                                                                                                                                                                                                                                                                                                                       110
                                                                                                                701
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                                   2222
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```

```
233 X=(2.*RADIUS-SQRT(LAMBDA**2.*(1.-E)+4.*E**2.*RADIUS**2.))/(2.*(1.-
                I'N EACH SUB ARRAY
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                NO 430 LOOF CALCULATES FIELD FOR THETA=M DEGREES
             ELÉMENTS
                                                                                                                                                                                                                                                                                                                                                                   ALPHA=-ATAN2(-X,SORT((RADIUS**2-X**2)/E))
                                                                                                                                                                                                                                                                                                                                                    IF (TARC-SUMARC-LAMBDA/2.) 205,205,294
               n
L
               NUMBER
                                                                                                                                                                                          EF(NP(MM)-NP(MM-1))240,241,241
                             THEM
                                                                                                                                                                                                                                                                                                                                                                                    X1=X-(LAMEDA/2.) *COS(ALPHA)
              THROUGH 205 - CALCULATES AND ARC LENGTH BETHEEN
                                                                                                                                                                                                                                        X=NP(MM)*LAMBDA/(2.*PI)
                                                                                                                                                                                                                                                                                                      ARC (MH) = ARC (MH) +H*F(Y)
                                                                                                                                                                                                                                                                                                                                      SUMARC=SUMARC+ARC (MM)
                                                                                                                                                                         IF (MM-1) 240, 240, 239
                                                                                                                                           NP(MM)=FI*X/LAMBDA
                                                                                                                                                                                                         NP (MM) = NP (MM-1) -2
                                                                                                                                                          NP (MM) = S*NP (MM)
                                                                                                                                                                                                                                                                                       or 115 II=1,100
                                                                                                                                                                                                                         (HH) dア+FON=HON
                                                                                                                                                                                                                                                        H=(X2-X)/100.
                                                                                                                          X2=RADIUS
                                                                                                                                                                                                                                                                                                                                                                                                                                                   60 TO 207
                                                                                                                                                                                                                                                                         Y=X+H/2.
                                                                                                                                                                                                                                                                                                                                                                                                                                                                  CONTINUE
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 MMA-NH+1
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  HYDXHXH
                                                                                                                                                                                                                                                                                                                                                                                                                                    NN=NK+1
                                                                                                                                                                                                                                                                                                                     Y=Y+H
                                                              N-HON
                                                                             XX=2
                                                                                                                                                                                                                                                                                                                                                                                                     X=CX
                                                                                                                                                                                                                                                                                                                                                                                                                    X=X1
                                                                                                                                                                                          239
241
240
                                                                                                                                                                                                                                                                                                                     115
                                                                                                                                                                                                                                                                                                                                                                      204
                                                                                                                                                                                                                                                                                                                                                                                                                                                                 202
                                                                                                                                           297
                            0000
0000
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0000
0000
             2222
5000
```

```
DO 250 LOOP SUMS FIELD CONTRIBUTIONS FROM EACH SUB
                                                                                                                                                                                                                                                                                                                                          DO 200 LOOP CALCULATES FIELD FROM EACH SUB ARRAY
DO 400 M=1<sub>8</sub>91
S7=SIN(THETAO/RAD)
                            C7=CnS(THETA0/RAD)
                                                                                                                                                                                                                                                                                                                                                                          DO 260 I=1,N
PHI=2.*PI*(I-1)/N
                                                                         C4=COS (THETA/RAD)
                                                                                        S4=SIN (THE TA/RAD)
                                                                                                                                                                                                                                 DO 250 MM=1, MMAX
                                                                                                                                                                                                                                                                              S2=SIN (THETA1)
                                                                                                                                                                                                                                                                                              C2=COS(THETA1)
                                                                                                                                                                                                                                                 S1=SIN (ALPHA)
                                                                                                                                                                                                                                                               C1=COS (ALPHA)
                                                          C5=COS (PHIO)
                                            (OIHd) NIS=SS
                                                                                                                                      A(M)=(0.,0.)
ALPHA=PI/2.
                                                                                                                                                                    THETA1=PI/2
                                                                                                                                                                                                                                                                                                                                                                                                       S3=SIN (PHI)
                                                                                                         X=RADIUS
                                                                                                                                                                                                                                                                                                             N=NP (MM)
                                                                                                                         71=0.
                                                                                                                                                                                                                                                                                                                            0000
0000
0000
                                                                                                                                                                                    0000
0000
0000
```

ARRAY

CÉ=COS (PHI1-PHI) S8=SIM (PHIO-PHI)

(IHd-DIHd) S00=80

S6=SIN (PHI1-PHI)

(1Hd) S00=20

```
DIST=SQRT((R*S4*C5-X*C3)**2+(R*S4*S5-X*S3)**2+(R*C4-71)**2)
                                                                                                                                                                              ELTFAC=SIN(1.6+PI+SIN(BETA)/2.)/(1.6+PI+SIN(BETA)/2.)
                                       9ETA=ACOS((S1*(R*S4*C8-X)+C1*(R*C4-Z1))/DIST)
                                                                                                                                                                                                                                                                                                                                                                                                APHASE (M) = ATAN2 (AIMAG (A (M)), REAL (A·(M))) *RAD
                                                                                                                                                                                                                                                                                                                                     ALPHA=-ATAN2 (-X, SORT ( (RADIUS ** 2 = X * + 2) /E))
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  FIELD AND ANGLE
                                                                                                                                                                                                                                                                                                                                                                                                                   700, THETA, A (M), AMAG (M), APHASE (M)
                                                                                                                                                                                                                                                                                                                                                                                                                                      PUNCH 837, THETA, A (M) & BRAG (M), APHASE (M)
                                                                                                                                                                                                                                                                              A (M) = A (M) + CEXP (RANGE) * PHASE * ELIFAC
PSI=-K*(21*C7-R401US*S7+X*S3*S7)
                                                                                                                                                                                                                                                                                                                   Z1=SORT ((RADIUS**2.-X**2.)*E)
                                                                              IF (RETA-PI/2.) 209,232,232
                                                                                                                                                                                                                                                                                                 X=NP(MM+1)*LAMBDA/(2.*PI)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               IF (AMAG(J)-0)401,402,401
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  - FINDS MAX
                                                          IF (BETA) 231,209,231
                                                                                                                                                                                                                                                                                                                                                          THET A1=ATAN2 (X, Z1)
                                                                                                                                                                                                                                                                                                                                                                              BHAG (M) = CABS (A (M))
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      D=AMAX1(D,AMAG(J))
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        PPINT 801,0,THFTA
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     THETA=FLOAT (JJ-1)
                                                                                                                                                                                                H=(0.,-1.) *DIST
                                                                                                                                                                                                                                                                                                                                                                                                                                                          THETA=THETA+1.
                                                                                                                                                                                                                     Z=(0,1,)*PSI
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   30 300 J=2,91
                                                                                                                                                                                                                                                           PHASE=CEXP(2)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          DO 461 J=1,91
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  THROUGH 462
                                                                                                                                                                                                                                       RANGE=K*X
                                                                                                ELTFAC=0.
                                                                                                                    GO TO 211
                                                                                                                                       ELTFAC=1.
                                                                                                                                                         GO TO 211
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 D=AMAG(1)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 CONTINUE
                                                                                                                                                                                                                                                                                                                                                                                                                   PRINT
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             13=3
                                                                                                                                        208
                                                                                                                                                                              239
211
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PEINT 805 00 900 I=1, MMAX 900 PEINT 804, I, NP(I) PFINT 804, MMA, MI PRINT 806, NUM GO TO 202 503 STOP END

TWO (INPUT, OUTPUT, PUNCH)

PROGRAM

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FORMAT (1H1, *SPACING *, I3, */16 WAVELENGTH*, 4X, *RADIUS*, F6.2, * METER
                                                                                                                                    OF ARRAY=
                                                                                                                                                                                                                                     ** CIRCLE
                                                                                                                                                                                     FORMAT(1H ,*BEAM COORDINATES--THETA= *,F3.0,2X,*PHI= *,F3.0,/)
                                                      F(X)=SQRT((RADIUS**2.-X**2.+E*X**2.)/(RADIUS**2.-X**2.))
                                                                                                                                                                                                                                 IN ARRAY ######*
                                                                                                                               CS*,4X,F4.1,* GHZ*,4X,FHI=*,F6.3,* CEGREES*,4X,*HEIGHI
                                  DIMENSION A (91), AMAG (91), APHASE (01), NF (500), ARC (500)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       8=INCREMENTAL POSITION OF FIELD POINT (IN PI/N STEPS)
                                                                                                                                                                        AT *,F3.0,*DEGREES*,/)
                                                                                                                                                                                                                                                                      IN ARRAY--*, IB)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         THETAO AND PHIP ARE BEAM COORDINATES IN DEGREES
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     H-HUMBER OF ELEMENTS AROUND BASE OF AFRAY
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              E1=RATIO OF HEIGHT OF ARRAY TO RADIUS CF
                                                                                                                                                                                                                                                                                                                                              TO PE USED IN PROGRAM
                                                                                                                                                                                                                              FORMAT (1H0, *TOTAL NUMBER OF ELEMENTS
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               THETA=START ANGLE FOR CALCULATIONS
                                                                                                                                                                    FORMAT (1H0, * MAXIMUM *, F15, 5, *
                                                                                           FORMAT (1H' , F6.1, 3F15.5, F6.0)
                                                                        FCPMAT (215,3F5.0,F5.3,F3.0)
 PANGE, PHASE, A, W, Z
                                                                                                                                                                                                                                                                                                     FORMAT (F5.3,3F15.5,2F3.0)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                           Ka2.*PI*FREQ/(3.*10.**8.)
GFREQ=FREQ/(10.**9.)
                                                                                                                                                                                                                                                                                     FORMAT (F6.1,3F15,5,F6.0)
                                                                                                                                                                                                                                                                                                                                                                                                                                                           L.AMBDA=3.*10.**8./FREQ
                                                                                                                                                                                                           ,16,9X,15)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   BEGIN CALCULATIONS
                                                                                                                                                                                                                                                                                                                                            DEFINES CONSTANTS
                                                                                                                                                                                                                                                CUMBER*, 4X, *NUMBER
                                                                                                                                                    BASE*)
                                                                                                                                                                                                                                                                                                                                                                                                                                       RAD=350./(2.*PI)
                                                                                                                                                                                                                                                                                                                                                                                                                      PI=4.*ATAN (1.0)
                                                                                                                                                                                                                                                                                                                                                                                 FREQ=2.*10.**9,
                REAL LAMBDA,K
                                                                                                                                                  C, F5.3,* OF
                                                                                                                                                                                                                              FORMAT (1H
                                                                                                                                                                                                           FORMAT (1H
COMPLEX
                                                                                                                                                                                                                                                                                                                                                                                                    PSI=0.
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CALCULATES REMAINING CONSTANTS DEPENDENT ON INPUT VARIABLES
                                                                                                                                                                                                                                                                                                                                                      CALCULATES ARC LENGTH FROM BASE OF ARRAY TO TOP
        READ 1,00,N,L,B,THETAO,PHIP,E1,THETA
                                              THROUGH 702 - INITIALIZES ARRAYS
                                                                                                                                                                                                                                                                                                                                                                                                                                       800, L, PADIUS, GFREQ, PH, E1
                                                                                                                                                                                                                                                                                          RADIUS=N#LAMBDA*L/(32.*PI)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                 BOB, EL, N, THETAD, PH
                                                                                                                                                                                                                                                                                                                                                                                                                                                     803, THETAD, FHIP
                                                                                                                                                       AMAG(I)=APHASE(I)=0
                                                                                                                                                                                                                                                                               PHI1=PI/2.-PHIF/RAD
                                                                                                                                                                                                                                                  PHIO=PI*8/N+PI/2.
                     IF (N) 503,503,592
                                                                                                                                                                                                                                                                                                                                                                                                                TARC=TARC+H*F(X)
                                                                                                                                                                                                                                                                                                                                                                                                   00 110 I=1,1000
                                                                                                                                                                                                                                                                                                          H=RADIUS/1388.
                                                                                       00: 701 I=1,500
                                                                                                                                                                                                                                                                                                                                   R=200.*RADIUS
                                                                                                                             DO 702 I=1,91
                                                                                                                                           A(I)=(0.,0.)
                                                                                                                                                                                                                                                                    PH=PHIO*RAD
                                                                          SUMARC=9.
                                                                                                      ARC(I)=0.
                                                                                                                                                                                                                                      NP(1)=N
                                                                                                                 NP(I)=0
                                                                                                                                                                                                            E=E1**2
                                                                                                                                                                                                                                                                                                                                                                                       TARC=0.
                                                                                                                                                                                                                                                                                                                       X=H/2.
                                                                                                                                                                                                                                                                                                                                                                                                                                                      PRINT
                                                                                                                                                                                                                                                                                                                                                                                                                                                                     PUNCH
                                                                                                                                                                                                                                                                                                                                                                                                                              H+X=X
                                                                                                                                                                                                                                                                                                                                                                                                                                           TNIdd
                                                                                                                                                                                                                         M1=1
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X=(2.*RADIUS-SQRT(LAMBDA**2.*(1.-E)+4.*E**2.*RADIUS**2.))/(2.*(1.-
                ELEMENTS IN EACH SUB ARRAY
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           00 460 LOOP CALCULATES FIELD FOR THETA=M DEGREES
                                                                                                                                                                                                                                                                                                                                                                                       234. ALPHA=-ATAN2 (-X, SORT ( (RADIUS**2-X**2) /E)
                                                                                                                                                                                                                                                                                                                                                                        IF (TARC-SUMARC-LARBDA/2.) 205,205,204
                THROUGH 205 - CALCULATES NUMBER OF AND ARC LENGTH BETHEEN THEM
                                                                                                                                                                                                    IF (NP (MM) - NP (MM-1)) 240,241,241
                                                                                                                                                                                                                                                                                                                                                                                                         X1=X-(LAMBDA/2.) *COS(ALPHA)
                                                                                                                                                                                                                                                      X=NP(MM)*LAMBDA/(2.*PI)
                                                                                                                                                                                                                                                                                                                       ARC (MM) = ARC (MM) +H*F(Y)
                                                                                                                                                                                                                                                                                                                                                        SUMARC=SUMARC+ARC (MM)
                                                                                                                                                                                    IF (MM-1) 240, 240, 239
                                                                                                                                                   NP(HM)=PI*X/LAMEDA
                                                                                                                                                                                                                     NP (MH) =NP (MM-1)-2
                                                                                                                                                                   NP (MM) = 5 * NP (MM) dN
                                                                                                                                                                                                                                                                                                       00 115 II=1,100
                                                                                                                                                                                                                                      NUX=NUX+NP (MX)
                                                                                                                                                                                                                                                                        H = (X2 - X) / 100
                                                                                                                                  X2=RADIUS
                                                                                                                                                                                                                                                                                                                                                                                                                                                                          GO TO 207
                                                                                                                                                                                                                                                                                        Y=X+H/2.
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           CONTINUE
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           MAG=NH+1
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            HWAX=MM
                                                                                                                                                                                                                                                                                                                                                                                                                                                           MM=MM+1
                                                                                                                                                                                                                                                                                                                                         H+X=X
                                                                  NUK=N
                                                                                  MM=2
                                                                                                                                                                                                                                                                                                                                                                                                                          X2=X
                                                                                                                                                                                                                                                                                                                                                                                                                                          X=X1
                                                                                                                    CE))
                                                                                                  233
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DO 250 LOOP SUMS FIELD CONTRIBUTIONS FROM EACH SUB ARRAY
                                                                                                                                                                                                                                                                                                               DO 200 LOOP CALCULATES FIELD FPOM EACH SUB ARRAY
DO 400 M=1,91
S7=SIN(THFTA0/RAD)
                            C7=COS(THETAD/RAD)
                                                                                                                                                                                                                                                                                                                                                 00 260 I=1.9N
PHI=2.*PI*(I-1)/N
                                                                     C4=COS (THE TA/RAD)
                                                                                   S4=SIN(THETA/RAD)
                                                                                                                                                                                                                                                                                                                                                                                                                       C6=COS (PHIA-PHI)
                                                                                                                                                                                                                 DO 250 MM=1, MMAX
                                                                                                                                                                                                                                                                                                                                                                                                         S6=SIR(PHI1-PHI)
                                                                                                                                                                                                                                                                                                                                                                                                                                    Se=SIN (PHIN-PHI)
                                                                                                                                                                                                                                                           S2=SIN (THETA1)
                                                                                                                                                                                                                                                                         C2=COS (THE TA1)
                                                                                                                                                                                                                                S1=SIN (ALPHA)
                                                                                                                                                                                                                                               C1=COS (ALPHA)
                                                                                                                              A(M)=(0.,0.)
ALPHA=PI/2.
                                                                                                                                                         THETA1=PI/2.
                                          (DIHA) NIS=SS
                                                        C5=COS (PHIO)
                                                                                                                                                                                                                                                                                                                                                                            S3=SIN (PHI)
                                                                                                                                                                                                                                                                                                                                                                                           C3=COS(PHI)
                                                                                                                                                                                                                                                                                         N=NP (MM)
                                                                                                  RADIUS
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CB=COS (PHIC-PHI)

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ELTFAG=SIN(1.6*PI*SIN(BETA)/2.)/(1.6*PI*SIN(BETA)/2.)
                                   BETA=ACOS((S1*(R*S4*C8+X)+C1*(R*C4+Z1))/OIST)
                                                                                                                                                                                                                                                                                                                                                                                APHASE(M)=ATAN2(AIMAG(A(M)), REAL(A(M))) *RAD
                                                                                                                                                                                                                                                                                                                     ALPHA=-AJAN2 (-X, SQRT ((RADIUS**2-X**2)/E)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                            THROUGH 402 - FINDS MAX FIELD AND ANGLE
                                                                                                                                                                                                                                                                                                                                                                                                 PPINT 700, THETA, A (M), AMAG (M), APHASE (N) PUNCH 807, THETA, A (M), AMAG (M), APHASE (M)
                                                                                                                                                                                                                                                                 A (M) = A (M) + CEXP (RANGE) * PHASE * EL TFAC
PSI=-K*(21*C7-RADIUS*C7+X*S3*S7)
                                                                                                                                                                                                                                                                                                     Z1=SQRT((RADIUS**2.-X**2.)*E)
                                                                        IF(BFTA-PI/2.)209,232,232
                                                                                                                                                                                                                                                                                    X=NP (MM+1) *LAMBDA/(2.*PI)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                IF (AMAG(J)-0)401,462,401
                                                     IF (BETA) 231,208,231
                                                                                                                                                                                                                                                                                                                                           THETA1=ATAN2(X,Z1)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      D- 366 J=2,91
D=AMAX1(D,AMAG(J))
                                                                                                                                                                                                                                                                                                                                                              AMAG (M) = CABS (A(M))
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      PRINT. 881, 0, THETA
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     THETA=FLOAT(JJ-1)
                                                                                                                                                                                      H= (0.,-1.) *DIST
                                                                                                                                                                                                                                                                                                                                                                                                                                        THETA=THETA+0:1
                                                                                                                                                                                                         Z=(0.,1.)*PSI
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           DC 401 J=1,91
                                                                                                                                                                                                                                               PHASE=CEXP(Z)
                                                                                                                                                                                                                            RANGE=K*H
                                                                                          EL TFAC=0.
                                                                                                                                                 60 TO 211
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     D=AMAG(1)
                                                                                                              60 TO 211
                                                                                                                               ELTFAC=1.
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 CONTINUE
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PRINT 805 00 900 I=1,MMAX 900 PFINT 804,I,NP(I) PFINT 804,MMA,MI PRINT 806,NUM GO TO 202 503 STOP END

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FORMAT (1H , F5.1,3F15.5,F6.0)
FORMAT (1H1,*SPACING *,I3,*/16 HAVELENGTH*,4X,*RADIUS*,F6.2,* METER CS*,4X,F4.1,* GHZ*,4X,*PHI=*,F6.3,* DEGREES*,4X,*HEIGHT OF ARRAY= *
                                                                                                                                                                                                                                                          ********* NUMBER OF ELEMENTS IN ARRAY #####*, * CIRCLE *NUMBER OF ELEMENTS*).
                                                                                                                                                                                                                      *,F3.0,/)
                                                                                                                                                                                                                                                                                                                                                               FOR FHI OF
                                                                            F(X)=SORT(PADIUS**2.-X**2.+E*X**2.)/(RADIUS**2.-X**2.)
                                                         A (91), AMAG (91), APHASE (91), NF (520), ARC (500)
                                                                                                                                                                                                                   ** BEAM CCORDINATES -- THETA = * F3.0, 2X, *PHI =
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                B=INCREMENTAL FOSITION OF FIELD POINT (IN FILM STEPS)
                                                                                                                                                                                                 FORMAT(1HC, *MAXIMUK *, F15.5, * AT *, F3.0, *DEGREES*, /)
                                                                                                                                                                                                                                                                                                    IN ARRAY--*, 18)
                                                                                                                                                                                                                                                                                                                                                               w
                                                                                                                                                                                                                                                                                                                                                             VARIATION IN
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    THETAD AND PHIP ARE BEAM COORDINATES IN DEGREES
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          M-NUMPER OF ELEMENTS AROUND BASE OF ARRAY
                                                                                                                                                                                                                                                                                                                                                                                                                         IN PROCRAM
                                                                                                                                                                                                                                                                                                   ELEMENTS
                                                                                                                                                                                                                                                                                                                                                             THE
THREE (INPUT, OUTFUT, PUNCH)
                                                                                                                                                                                                                                                                                                                                                             CALCULATES
                                                                                                                                                                                                                                                                                                                                                                                                                         BE USED
                                                                                                                                                                                                                                                                                                  NUMPER CF
                                                                                                FORMAT (215,3FF.0,FF.3,F3.0)
                   RANGE, PHASE, 4, H, Z
                                                                                                                                                                                                                                                                                                                                         FORMAT (F5.3,3F15.5,2F3.0)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               K=2.*PI*FREG/(3.*10.**8.)
                                                                                                                                                                                                                                                                                                                   FORMAT (F5.1,3F15.5,F6.0)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            LAMBOA=3.*10.**8./FREG
                                                                                                                                                                                                                                                                                                                                                               とりだ
                                                                                                                                                                                                                                                                                                                                                                                                                        DEFINES CONSTANTS TO
                                                                                                                                                                                                                                         ,I6,9X,I5)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    GFREQ=FRE0/(10.**9.)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           BEGIN CALCULATIONS
                                                                                                                                                                                                                                                                                CUMBER*, 4X, *NUMBER
                                                                                                                                                                                                                                                                                                   FORMAT (1H9, *TOTAL
                                                                                                                                                                              BASE*)
                                                                                                                                                                                                                                                                                                                                                             FORMAT (1H .*THIS
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         RAD=350./(2.*PI)
                                                                                                                                                                                                                                                                                                                                                                                                                                                               FFE0=2.*10.**9.
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       PI=4.*ATAN (1.9)
                                       REAL LAMBDA,K
                                                          DIMENSION
                                                                                                                                                                                                                                          FORMAT (1H
                                                                                                                                                                                                                                                             FORMAT (1H
                                                                                                                                                                                                                        FORMAT (1H
PROGRAM
                    COMPLEX
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  PSI=0.
                                                                                                                                                                                                                                                             805
                                                                                                                                                                                                                   803
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CALCULATES REMAINING CONSTANTS DEFENDENT ON INPUT VARIABLES
                                                                                                                                                                                                                                                                                                                                                                                                                 CALCULATES ARC LENGTH FROM BASE OF ARRAY TO TOP
E1=RATIO OF HEIGHT OF ARRAY TO RADIÚS OF, BASE
THETA=START ANGLE FOR CALCULATIONS
                                          PEAD 100,N,L,B,THETAD,PHIP,E1,THETA.
IF(N)503,503,502
                                                                                    THROUGH 702 - INITIALIZES ARRAYS
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   PPINT 803.L.RADIUS,GFREQ,PH,E1
                                                                                                                                                                                                                                                                                                                                             RADIUS=N*LAMBDA*L/(32.*PI)
                                                                                                                                                                                    A(I)=(0.90.)
AHAG(I)=APHASE(I)=0.
                                                                                                                                                                                                                                                                                                                                PHI1=PI/2.-PHIP/RAD
                                                                                                                                                                                                                                                                                                   PHIO=PI#B/N+FI/2.
                                                                                                                                                                                                                                                                                                                                                                                                                                                                         TARC=TARC+H*F(X)
                                                                                                                                                                                                                                                                                                                                                                                                                                                          00 11C I=1,1000
                                                                                                                             DO 761 I=1,500
                                                                                                                                                                                                                                                                                                                                                           H=RADIUS/1009.
                                                                                                                                                                    DO 702 1=1,91
                                                                                                                                                                                                                                                                                                                                                                                       R=200.*PADIUS
                                                                                                                                                                                                                                                                                                                PH=PHIO*2AD
                                                                                                                                          APC(I)=0.
                                                                                                               SUMAPC=0.
                                                                                                                                                         NP(I)=0
                                                                                                                                                                                                                                                                                      N=(1)=N
                                                                                                                                                                                                                                                          E=E1**2
                                                                                                                                                                                                                                                                                                                                                                                                                                             TARC=0.
                                                                                                                                                                                                                                                                                                                                                                         X=H/2.
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       H+X=X
                                                                                                                                                                                                                                                                        H1=1
                                           202
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X=(2.*RADIUS-SURT(LAMBDA**2.*(1.-E) +4.*E**2.*RADIUS**2.))/02.*(1.-
                                                        OF ELEMENTS IN EACH SUB ARRAN
                                                                                                                                                                                                                                                                                                                                                                             ALPHA=-ATAN2(-X,SORT((RADIUS**2-X**2)/E))
                                                                                                                                                                                                                                                                                                                                                               IF (TARC-SUMARG-LANBDA/2.) 205, 205, 204
                                                        NUMBER
                                                                                                                                                                                                                   IF(NP(HM)-NP(KM-1))200,241,241
                                                                      THEM
                                                                                                                                                                                                                                                                                                                                                                                            X1=X-(LAM6DA/2.) *COS(ALPHA)
                                                                     AND ARC LENGTH PETHEEN
                                                       THROUGH 205 - CALCULATES
                             808, E1, N, THETAD, PH
                                                                                                                                                                                                                                                              X=NP (KK) *L ÅKBOA/ (2. *PI)
                                                                                                                                                                                                                                                                                                       90 115 II=1,100
ARC:(MM)=ARC(MM)字样*序(Y)
803, THETAO, PHIP
                                                                                                                                                                                                                                                                                                                                                  SUMARC=SUMAFC+ARC (MM)
                                                                                                                                                                                                    IF (HM-1)240,240,239
                                                                                                                                                                       NETHEN = PI*X/LAMBDA
                                                                                                                                                                                                                                  NP (MH) = NP (PH-1) +2
                                                                                                                                                                                       NP (HH) = 2*NP(HH)
                                                                                                                                                                                                                                                NUM-NUM+ND (MM)
                                                                                                                                                                                                                                                                           H=(X2-X)/106°
              666
                                                                                                                                                           X2=PADIUS
                                                                                                                                                                                                                                                                                                                                                                                                                                                      50 TO 207
                                                                                                                                                                                                                                                                                           Y=X+H/2.
                                                                                                                                                                                                                                                                                                                                                                                                                                                                    CONTINUE
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                HKAX=MM
                                                                                                                                                                                                                                                                                                                                                                                                                                       HM=HH+1
                            PUNCH
PPINT
                                                                                                                                                                                                                                                                                                                                     ドナ人=人
             PRINT
                                                                                                   NUH
                                                                                                                                                                                                                                                                                                                                                                                                           X 5=X
                                                                                                                                                                                                                                                                                                                                                                                                                         X=X1
                                                                                                                 MMII 2
                                                                                                                               203
                                                                                                                                                                                                                                                                                                                                                                                                                                                                    225
                                                                                                                                                                                                                   233
241
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DO 250 LOOP SUMS FIELD CONTRIBUTIONS FROM EACH SUB ARRAY
          DO 400 LOOP CALCULATES FIFLD FOR THETA=M DEGREES
                                                                                                                                                                                                                                                                                                                                                                                                   DO 200 LOOF CALCULATES FIELD FROM EACH SUB ÁRRAY
                                                                          S7=SIN(THETAD/RAB)
                                                                                        C7=COS (THE TAG/RAD)
                                                                                                                                                                                                                                                                                                                                                                                                                                  50 260 I=1,N
PHI=2.*PI*(I-1)/N
                                                                                                                                       C4=COS (THETA/RAD)
                                                                                                                                                     S4=SIN(THETA/RAD)
                                                                                                                                                                                                                                                                                            DO 250 HM=1, HHAX
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              S6=SIN (PHI1-PHI)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             CE=COS (PHI1-PHI)
                                                                                                                                                                                                                                                                                                                                         SP=SIN (THETA1)
                                                                                                                                                                                                                                                                                                                                                        C2=COS (THETA1)
                                                                                                                                                                                                                                                                                                          S1=SIN(ALPHA)
                                            00 400 M=19:0
                                                                                                                                                                                                                                                                                                                         C1=COS (ALPHA)
                                                                                                                                                                                                  A(M)=(0.,0.)
ALPHA=PI/2.
                                                                                                                                                                                                                               THETA1=PI/2.
                                                                                                                       C5=COS (PHIO)
                                                                                                        SE-SIN(PHIO)
                                                            THETAS=THET
                                                                                                                                                                                                                                                                                                                                                                                                                                                                SZ=SIN(PHI)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                C3=COS(PHI)
                                                                                                                                                                    X=RADIUS
                                                                                                                                                                                                                                                                                                                                                                        N=NP(MM)
                                                                                                                                                                                   21=0.
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S8=SIN(PHIO+PHI) C8=COS(PHIO-PHI)

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DIST=SQRT((R*::4*C5-X*f3)**2+(R*S4*S5-X*S3)**2+(R*64-Z1)**2)
                                                                                                                                                                               ELTFAC=SIN(1.6*PI*SIN(BETA)/2.)/(1.6*PI*SIN(BETA)/2.)
                                        PETA=ACOS((S1*(R*S4*C3-x)+C1*(R*C4-21))/DIST)
                                                                                                                                                                                                                                                                                                                                                                                                   APHASE (M)=ATAN2 (AIMAG (A(M)), REAL (A(M))) *RAD
                                                                                                                                                                                                                                                                                                                                       ALPHA=-ATAN2 (-X, SORT ( (RADIUS**2-X**2)/E)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           - FINDS MAX FIELD AND ANGLE
                                                                                                                                                                                                                                                                                                                                                                                                                                          PPINT 700, PH, A(M), AMAG(M), APHASE(M)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                PUNCH 807, PH, A(P), AMAG(M), APHASE(R)
                                                                                                                                                                                                                                                                               A (H) = A (H) + CEXP (RANGE) * PHASE* ELTFAC
PSI=-K*(Z1*C7-RADIUS*S7+X*S3*S7)
                                                                                                                                                                                                                                                                                                                      Z1=SORT ((RADIUS**2.-X**2.\*E)
                                                                             IF (BETA-P1/2.) 209 . 3: 4232
                                                                                                                                                                                                                                                                                                   K=NP (MH+1) *LAMBDA/(2.*PT)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  IF (AMAG(J)-0)401,462,401
                                                         IF (RETA) 231, 208,2 *4
                                                                                                                                                                                                                                                                                                                                                          THETA1=ATAN2 (X, Z1)
                                                                                                                                                                                                                                                                                                                                                                                AMAG (M) =CABS (A.(H))
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          D=AMAX1(C,AMAG(J))
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      PHIO=PHIO+1.0/RAD
                                                                                                                                                                                                    W=(0.,-1.)*DIST
                                                                                                                                                                                                                       ISd*(0,00)=Z
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           00 401 J=1,91
JJ=J
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     00 30C J=2,91
                                                                                                                                                                                                                                                            PHASE=CEXP (Z)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              THROUGH 402
                                                                                                                                                                                                                                                                                                                                                                                                                       PH=PHIO*RAC
                                                                                                                                                                                                                                         RANGEHK*
                                                                                                 ELTFAC=9.
                                                                                                                     GO TO 211
                                                                                                                                        ELTFAC=1.
                                                                                                                                                            GO TO 211
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    D=AMAG(1)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      CONTINUE
                                                                                                                                         2º8
                                                                                                                                                                                233
211
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       0 C 7
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()
                                                                              232
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                                                                                                                                                                                                                                                                               200
                                                                                                                                                                                                                                                                                                                                                            250
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432 THETA=FLOAT(JJ-1)
PRINT 801,D, THETA
PRINT 805
DO 900 I=1,MMAX
900 PRINT 804, I,NP(I)
PRINT 804,MMA,M1
PRINT 805,NUM
GO TO 202
503 STOP
END

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| FORMAT(1H1, *SFACING *, I3, */146, WAVELENGTH*, 4x, *RADIUS*, F6.2, * METER CS*, 4x, F4.1, * GHZ*, 4x, *FHI=*, F6.3, * CEGREES*, 4x, *HEIGHT CF ARRAY= *
                                                                                                                                                                                                                                                      CIRCLE
                                                                                                                                                                                                                *, F3.09/)
                                                                         F(X)=SQ&T((RADIUS**2.-X**2.+E*X**2.)/(RADIUS**2.-X**2.))
                                                                                                                                                                                                                                                   ARRAY #####*
                                                       DIMENSION A (91), AMAG (91), APHASE (91), NF (500), ARC (500)
                                                                                                                                                                                                             *,F3.0,2X,*PHI=
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                8=INCREMENTAL FOSITION OF FIELD PCINT (IN PAZN STEPS)
                                                                                                                                                                                            *,F3.0, CEGREES*,/)
                                                                                                                                                                                                                                                                                          ARRAY -- *, I8)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 THFTAO AND PHIP ARE GRAM COORDINATES IN DEGREES
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             N=NUMBER OF ELEMENTS AROUND BASE OF ARRAY
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    FI=RATIO OF HEIGHT OF ARRAY TO RADIUS OF
                                                                                                                                                                                                                                                    Z
                                                                                                                                                                                                                                                                                           Z
                                                                                                                                                                                                                                                                                                                                                                  SE USED IN PROGRAM
                                                                                                                                                                                                          **BEAM COGROINATES--THETA=
                                                                                                                                                                                                                                                  **##### NUMBER OF ELEMENTS
                                                                                                                                                                                                                                                                                      NUMBER OF ELEMENTS
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         CALCULATIONS
                                                                                                                                                                                            77
FOUR (INPUT, OUTPUT, PUNCH)
                                                                                                                                                                                                                                                                   OF ELEMENTS*)
                                                                                                                                                                                        FORMAT (1H0, * HAXIPUM *, F15.5, *
                                                                                                             FORMAT (1H , F6.1, 3F15.5, F6.0)
                                                                                          FORMAT (215,3F5.3,F5.3)
                   RANGE, PHASE, A, W, Z
                                                                                                                                                                                                                                                                                                                            FORMAT (F5. 3, 3F15.5, 5, 2F3.0)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   K=2.*PI*FRE0/(3.*10.**8.)
                                                                                                                                                                                                                                                                                                          FORMAT (F6.1, 3F15.5, F6.0)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 LAMPDA=3.*13.**8./FREQ
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        ANGLE FOR
                                                                                                                                                                                                                                                                                                                                                                 DEFINES CONSTANTS TO
                                                                                                                                                                                                                              ,I6,9X,I5)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       GFREQ=FRE0/(10.**9.)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           BEGIN CALCULATIONS
                                                                                                                                                                                                                                                                                      FORMAT (1H3, *TOTAL
                                                                                                                                                                                                                                                                    CUMBER*, 4X, *NUMBER
                                                                                                                                                                       BASE*)
                                                                                                                                                                                                                                                                                                                                                                                                                                                               RAD=360./(2.*PI)
                                                                                                                                                                                                                                                                                                                                                                                                                                            PI=4.*ATAN (1.0)
                                                                                                                                                                                                                                                                                                                                                                                                      FRE0=2.*10.**9.
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         THET M = START
                                                                                                                                                                      C, F5.3,* OF
                                                                                                                                                                                                           FORMAT (1H
                                                                                                                                                                                                                                                 FORMAT (1H
                                                                                                                                                                                                                               FORMAT (1H
PROGRAM
                  COMPLEX
                                                                                                                                                                                                                                                                                                                                                                                                                         • 0=ISd
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CALCULATES REMAINING CONSTANTS DEPENDENT ON INPUT VARIABLES
                                                                                                                                                                                                                                                                                                                                                   CF ARRAY TO TOP
READ 130,N,L,B,THETAO,PHIP,E1,THETA
IF (N)503,503,502
                                       THROUGH 732 - INITIALIZES ARRAYS
                                                                                                                                                                                                                                                                                                                                                      BASE
                                                                                                                                                                                                                                                                                                                                                                                                                                    800, L, RADIUS, GFREQ, PH, E1
803, IHETAO, PHIP
                                                                                                                                                                                                                                                                                                                                                      CALCULATES ARC LENGTH FROM
                                                                                                                                                                                                                                                                                     RADIUS=N*LAMBDA*L/(32.*PI)
                                                                                                                                                                                                                                                                                                                                                                                                                                                              838, E1, N, THETAOSPH
                                                                                                                                               AMAG(I)=APHASE(I)=0.
                                                                                                                                                                                                                                                                        PHI1=PI/2.-PHIP/RAD
                                                                                                                                                                                                                                          F.:IO=PI*8/N+PI/2.
                                                                                                                                                                                                                                                                                                                                                                                                         TARC=TARC+F*F(X)
                                                                                                                                                                                                                                                                                                                                                                                             DC 110 I=1,1000
                                                                              00 701 I=1,530
                                                                                                                                                                                                                                                                                                 H=RADIUS/1009.
                                                                                                                    DO 702 I=1,91
                                                                                                                                                                                                                                                                                                                           R=200.*RADIUS
                                                                                                                                  A(I)=(0.,0.)
                                                                                                                                                                                                                                                            PH=PHIO*RAD
                                                                  SUMARC=0.
                                                                                          ARC(I)=0.
                                                                                                        NP(E)=0
                                                                                                                                                                                                    E=E1**2
                                                                                                                                                                                                                              N=(T)dN
                                                                                                                                                                                                                                                                                                                                                                                 TARC=0.
                                                                                                                                                                                                                                                                                                               X=H/2.
                                                                                                                                                                                                                                                                                                                                                                                                                                    +NIda
                                                                                                                                                                                                                                                                                                                                                                                                                                               PRINT
                                                                                                                                                                                                                                                                                                                                                                                                                                                              PUNCH
                                                                                                                                                                                                                                                                                                                                                                                                                        H+X=X
                                                                                                                                                                                                                 M1=1
252
                                                                  532
                                                                                                                                               792
                                                                                                         751
                                                                                                                                                                                                                                                                                                                                                                                                                        1100
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X=(2.*RADIUS-SORT(LAMBDA**2.*(1.~E)+4.*E**2.*RADIUS**2.))/(2.*(1
               ELEMENTS IN EACH SUB ARRAY
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      DO 400 LOOP CALCULATES FIELD FOR THETAHM DEGREES
                                                                                                                                                                                                                                                                                                                                                 ALPHA=-ATAN2 (-X, SCRT ((RADIUS**2-X**2)/E)
                                                                                                                                                                                                                                                                                                                                    IF (TARC-SUMA CC-LAMBOA/2.) 205, 205, 204
               NUMBER OF
                                                                                                                                                                                  IF (NF (MM) — NP (MM—1) 1,240,241,241
                             THEM
                                                                                                                                                                                                                                                                                                                                                                 X1=X-(LAMRBA/2.) *50S(ALPHA)
                - CALCULATES
                             AND ARG GENGTH BETWEEN
                                                                                                                                                                                                                            X=NP(MM)*LAMBDA/(2.*PI)
                                                                                                                                                                                                                                                                                       ARC (MM) = ARC (MM) + H*F (Y)
                                                                                                                                                                                                                                                                                                                     SUMARC=SUMARC+ARC (MM)
                                                                                                                                                                  IF (MM-1) 240, 240, 239
                                                                                                                                    NP (MM) = P.T.* X/LAMBDA
                                                                                                                                                                                              NP (MM)=NP (MM-1)-2
                                                                                                                                                                                                                                                                         00 115 II=1,100
                                                                                                                                                   NP(MM)=2*NP(MM)
                                                                                                                                                                                                             NUM=NUM+NP (MM)
                                                                                                                                                                                                                                           H=(X2-X)/1J9.
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   00 400 M=1,91
               THROUGH 205
                                                                                                                       X2=RADIUS
                                                                                                                                                                                                                                                                                                                                                                                                                           GO TO 237
                                                                                                                                                                                                                                                           Y=X+H/2.
                                                                                                                                                                                                                                                                                                                                                                                                                                           CONTINUE
                                                                                                                                                                                                                                                                                                                                                                                                                                                                         MMA=MM+1
                                                                                                                                                                                                                                                                                                                                                                                                                                                           ME-XAMM
                                                                                                                                                                                                                                                                                                                                                                                                              MK=MM+1
                                                                                                                                                                                                                                                                                                       N-MON
                                                                           アメニス
                                                                                                                                                                                                                                                                                                                                                                                X=2X
                                                                                                                                                                                                                                                                                                                                                                                                X=X1
                                                                                         203
                                                                                                                                                                                  239
                                                                                                                                                                                              241
                                                                                                                                                                                                                                                                                                       115
                                                                                                                                                                                                                                                                                                                                                   534
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                                                                                                                                     207
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0IST=SQ2T((R*S4*C5=X*C3)**2+(R*S4*S5=X*S3)**2+(R*C4=Z1)**2)
                                                                                                                                                                                                                                    DO 250 LOOP SUMS FIELD CONTRIBUTIONS FROM EACH SUB ARRAY
                                                                                                                                                                                                                                                                                                                                                                                               DO 200 LOOF CALCULATES FIELD FROM EACH SUB ARRAY
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            BETA=ACOS((S1*(R*S4*C8-X)+C1*(P*C4-Z1))/DIST)
IF(RETA)231,208,231
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              PSI=-K*(Z1*C7-R4GIUS*S7+X*S3*S7)
                     S7=SIN(THETAB/RAD)
                                       C7=COS(THETAD/RAD)
                                                                                                                                                                                                                                                                                                                                                                                                                                   00 200 I=1,N
PHI=2,*PI*(I-1)/N
                                                                                            C4=COS (THETA/RAD)
                                                                                                             S4=SIN(THE TA/RAB)
                                                                                                                                                                                                                                                                         250 MM=1, NHAX
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         C6=C0S(PHI1-PHI)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        S6=SIN(PHT1-FHI)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           SEEVIN (PHICEPHI)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            C8=CO~(PHIO-PHI)
                                                                                                                                                                                                                                                                                                                            S2=SIN (THE TA1)
                                                                                                                                                                                                                                                                                                                                            C2=COS (THETA1)
                                                                                                                                                                                                                                                                                         S1=SIN(ALPHA)
THETA 0=THETA
                                                                                                                                                                                                                                                                                                           C1=COS (ALPFA)
                                                           SE=SIN(PHIO)
                                                                           C5=C05 (PHIO)
                                                                                                                                                                   A (M)=(0.,0.)
                                                                                                                                                                                                    THETA1=PI/2
                                                                                                                                                                                  ALPHA=PI/2.
                                                                                                                                                                                                                                                                                                                                                                                                                                                                      (THA) NIS=ES
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        C3=COS (PHI)
                                                                                                                                X=RADIUS
                                                                                                                                                                                                                                                                                                                                                              N=NP(MM)
                                                                                                                                                  21=0.
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ELTFAG=SIN(1.6*PI*SIN(PETA)/2.)/(1.6*FI*SIN(BETA)/2.)
                                                                                                                                                                                                                                                                           APHASE (M)=ATAN2 (AIMAG (A(M)), REAL (A (Y))) *RAD
                                                                                                                                                                                                         Z1=SORT((RADIUS**2.-X**2.)*E)
ALPHA=-ATAN2(-X,SGPT((RADIUS**2-X**2)/E)
                                                                                                                                                                                                                                                                                                                                                                       - FINDS MAX FIELD AND ANGLE
                                                                                                                                                                                                                                                                                                            PRINT 739, FH, A(M), AMAG(M), APHASE(M)
PUNCH 837, FH, A(M), AMAS(P), APHASE(M)
                                                                                                                                                                        A (M).=A (M) +CEXP (RANGE) * PHASE*ELTFAC
IF (BETA-PI/2.)209,232,232
                                                                                                                                                                                         X=NP (MH+1) *LAMBOA7 (2. *PI)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  [F(AMAG(J)-D)491,402,401
                                                                                                                                                                                                                                       THETA1=ATAN2 (X, 21)
                                                                                                                                                                                                                                                            AMAG (H) = CABS (A (M))
                                                                                                                                                                                                                                                                                                                                                                                                                                                D=AMAX1(D,AMAG(J))
                                                                                                                                                                                                                                                                                                                                               PHIO=PHIO+0.1/RAD
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    PRINT 861,0,THETA
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  THETA=FLOAT(JJ-1)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    PPINT 834, I, NP(I)
                                                                                                     W=(0.,-1.)*DIST
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      DO 9Cc I=1, MMAX
                                                                                                                      =(00,10)*PSI
                                                                                                                                                                                                                                                                                                                                                                                                                              no 386 J=2,91
                                                                                                                                                                                                                                                                                                                                                                                                                                                                 00 461 J=1,91
                                                                                                                                                        PHASE=CEXP(Z)
                                                                                                                                                                                                                                                                                                                                                                              THROUGH 402
                                                                                                                                                                                                                                                                                             PH=PHIO*RAD
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    PPINT 835
                                                                                                                                       RANGE=K*4
                   ELTFAC=0.
                                   60 TO 211
                                                   ELTFAC=1.
                                                                     GO TO 211
                                                                                                                                                                                                                                                                                                                                                                                                                D=AMAG(1)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  CONTINUE
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     いこつつ
                                                   208
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                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   715
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                 232
                                                                                    239
                                                                                                                                                                        260
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PRINT 804, MMA, M1 PRINT 896, NUM GO TO 202 503 STOP END

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100 FORMAT (215,3F5.1,3F1E.5)F7.6)
700 FORMAT (1H ,F5.1,3F1E.5)F6.0)
800 FORMAT (1H1,*SPACING *, I3,*/16 MAVELENGTH*,4X,*RADIUS*,F6.2,* METER
CS*,4X,F4.1,* GHZ*,4X,*PHI=*,F5.3,* DEGREES*,4X,*HEIGHT OF ARRAY= *
                                                                                                                                                                                                **##### NUMPER OF ELEMENTS IN ARRAY #####*, / ,* CIRCLE
                                                                                                                                                                    *,F3.0.1)
                                   OIMENSION A(91), AMAS(91), APHACE(91), NP (500), ARC(500) F(X) = SART((RADIUS**2.-X**2.+E*X**2.)/(PADIUS**2.-X**2.))
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        N=HUMBER OF ELEMENTS AROUND BASE OF APRAY
B=INCREMENTAL POSITION OF FIELD POINT (IN PI/N STEPS)
                                                                                                                                                       FORMAT(1H, , *MAXIMUM *, F15.5, * AT *, F3.6, *DEGREES*, ?) FORMAT(1H, , *BEAM GOORDINATES--THETA= *, F3.0, 2X, *PHI=
                                                                                                                                                                                                                                     806 FORMAT (1H2, *TOTAL NUMBER OF ELEMENTS IN ARRAY -- *, 18)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          THETAG AND PHIP ARE BEAM CCORDINATES IN DEGREES
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            APRAY TO RADIUS OF BASE
                                                                                                                                                                                                                                                                                                             DEFINES CONSTANTS TO BE USED IN PROGRAM
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   THETA=START ANGLE FOR CALCULATIONS
                                                                                                                                                                                                                               CURBER*, 4X, *NUMBER OF ELEMENTS*)
BE AM (INPUT, CUT PUT, PUNCH)
                                                                                                                                                                                                                                                                                                                                                                                                                                      K=2.*PI*FREG/(3.*10.**8.)
                                                                                                                                                                                                                                                                                     FORMAT (F5. 3, 3F15.5, 2F3.0)
                COMPLEX PANGE, PHASE, A, H, Z
                                                                                                                                                                                                                                                                      ,3F15.5,F5.0)
                                                                                                                                                                                                                                                                                                                                                                                                                       LAMBDA=3.*10.**8./FREQ
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        E1=9ATIO OF HEIGHT OF
                                                                                                                                                                                                                                                                                                                                                                                                                                                        GFREQ=FREQ/(10.**3.)
                                                                                                                                                                                                      ,16,9X,15)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          BEGIN CALCULATIONS
                                                                                                                                                                                                                                                                                                                                                                                                            (Id**2)/*092=6Và
                                                                                                                                                                                                                                                                                                                                                                                            PI=4.*ATAN (1.0)
                                                                                                                                                                                                                                                                                                                                                           FRE0=2.*10.**9.
                                                                                                                                                                                                                                                                           FORMAT (F6.1
                                                                                                                                                          C, F5.3,* OF
                                                                                                                                                                                                                            805 FORMAT (1H
                                                                                                                                                                                                            804 FORMAT (1H
            PPOGRAM
                                                                                                                                                                                                                                                                                                 808
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   2000
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CALCULATES REMAINING CCNSTANTS DEPENDENT ON INPUT VARIABLES
                                                                                                                                                                                                                                                                                                                                                OF ARRAY TO TOP
READ 100,N,L,B,THETAO,PHIF,E1,THETA IF(N)503,503,502
                                      THROUGH 702 - INITIALIZES ARRAYS
                                                                                                                                                                                                                                                                                                                                                CALCULATES ARC LENGTH FROM BASE
                                                                                                                                                                                                                                                                                                                                                                                                                                800, L, RADIUS, GFREO, PH, E1
                                                                                                                                                                                                                                                                                  RADIUS=N*LAM3DA*L/(32.*PI)
                                                                                                                                                                                                                                                                                                                                                                                                                                                          858, E1, N, THETA3, PH
                                                                                                                                                                                                                                                                                                                                                                                                                                             883, THETAS, PHIP
                                                                                                                                                 AHAG (I) = APHASE (I) = 0.
                                                                                                                                                                                                                                                                     PHI1=PI/2.-PHIP/RAD
                                                                                                                                                                                                                                             PHI0=PI*B/N+PI/2.
                                                                                                                                                                                                                                                                                                                                                                                           DC 119 I=1,1900
TARC=TARC+H*F(X)
                                                                                                                                                                                                                                                                                                 H=RADIUS/1040.
                                                                               00 791 I=1,500
                                                                                                                                                                                                                                                                                                                          R=2:0.*RADIUS
                                                                                                                       no 702 I=1,91
                                                                                                                                     A(I)=(0.,0.)
                                                                                                                                                                                                                                                         PH=PHIO*RAD
                                                                                              ARC(I)=0.
                                                                    SUMARC=9.
                                                                                                           0=(I)dN
                                                                                                                                                                                                       E=E1**2
                                                                                                                                                                                                                                 N=(T) =N
                                                                                                                                                                                                                                                                                                                                                                               TAPC=0.
                                                                                                                                                                                                                                                                                                              X=H/2.
                                                                                                                                                                                                                                                                                                                                                                                                                                                             PUNCH
                                                                                                                                                                                                                                                                                                                                                                                                                       H+X=X
                                                                                                                                                                                                                                                                                                                                                                                                                                    PRINT
                                                                                                                                                                                                                                                                                                                                                                                                                                                PRINT
                                                                                                                                                                                                                   M1=1
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2.3 X=(2.*RADIUS-SQGT(LAMBDA**2.*(1.-E)+4.*E**2.*FADIUS**2.))/(2.*(1.-
ELEMENTS IN EACH SUB ARRAY
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         DO 4GG LOOP GALBULATES FIELD FOR THETA=M DEGREES
                                                                                                                                                                                                                                                                                                                                                          ALPHA=-ATAN2(-X,SORT((RADIUS**2-X**2)/E))
                                                                                                                                                                                                                                                                                                                                            IF (TARC-SUMARC-LAMEDA/2.) 205,205,204
 P
NUMBER
                                                                                                                                                                             IF (NP (MM) - NP (MM-1)) 240, 241, 241
                THEM
                                                                                                                                                                                                                                                                                                                                                                            X1=X-(LAMBDA/2.) *COS(ALPHA)
THROUGH 205 - CALCULATES
               AND ARC LENGTH BETHEEN
                                                                                                                                                                                                                             X=NP (MM) *LAMADA/ (2. *PI)
                                                                                                                                                                                                                                                                                            ARC (HH) = ARC (HH) +H*F (Y)
                                                                                                                                                                                                                                                                                                                             SUNA PC = SUM ARC + ARC (MM)
                                                                                                                                                              IF (MM-1)243,240,239
                                                                                                                               NF(MM) =PI * X/L AMBDA
                                                                                                                                                                                               NF (MM) = NP (MM-1) +2
                                                                                                                                                                                                                                                                              90 115 II=1,100
                                                                                                                                               NP (NW) = 2 * NP (NH)
                                                                                                                                                                                                              CHH) aN+WON=KON
                                                                                                                                                                                                                                              H=(X2-X)/100.
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           00 400 M=1,91
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           THETA0=THETA
                                                                                                                X2=PADIUS
                                                                                                                                                                                                                                                                                                                                                                                                                                            GC TO 297
                                                                                                                                                                                                                                                               Y=X+H/2.
                                                                                                                                                                                                                                                                                                                                                                                                                                                            CONTINUE
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             KMA=MM+1
                                                                                                                                                                                                                                                                                                                                                                                                                                                                             HYAX=MH
                                                                                                                                                                                                                                                                                                                                                                                                                            XX=KM+1
                                                 NIW-N
                                                                                                                                                                                                                                                                                                              Y=Y+H
                                                                NII NI
                                                                                                                                                                                                                                                                                                                                                                                            ジーでメ
                                                                                                                                                                                                                                                                                                                                                                                                            X=X1
                                                                                                CE)
                                                                                                                                                                              539
                                                                                                                                                                                                                                                                                                                                                             504
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PSI=-K*(Z1*C7-RADIUS*S7+X*S3*S7}.
DIST=SQRT((R*S4*C5-X*C3)**2+(R*S4*S5-X*S3)**2+(R*C4-Z1)*#2)
                                                                                                                                                                                                            DO 250 LOOP SUMS FIELD CONTRIBUTIONS FROM EACH SUB ARRAY
                                                                                                                                                                                                                                                                                                                                                                      DG 200 LOÓP CALCULATES FIELD FROM EACH SUM ARRAY
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      9EIA=ACOS((S1*(R*S4*C8-X)+C1*(R*C4-Z1))/DISI)
S7=SIN(THETAE/RAD)
                C7=COS(THETAD/RAD)
                                                                                                                                                                                                                                                                                                                                                                                                            00 200 I=1,N
PHI=2.*PI*(I-1)/N
                                                                    C4=COS (THE TA/RAJ)
S4=SIN (THE TA/PAD)
                                                                                                                                                                                                                                                 00 250 HM=1, MMAX
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   SE="IN (PHI1-PHI)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   C6=COS (PHI 1-PHI)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    (IHd-DIHd)NIS=2S
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      C8=COS(PHTO-PHT)
                                                                                                                                                                                                                                                                                                                      C2=COS (THETA1)
                                                                                                                                                                                                                                                                                                     S2=SIN(THETA1)
                                                                                                                                                                                                                                                                  S1=SIN (ALPHA)
                                                                                                                                                                                                                                                                                    C1=COS (ALPHA)
                                   SE-SIN(PHIO)
                                                     (0IHd) 500=50
                                                                                                                                                                             THETA1=PI/2
                                                                                                                                          A (M. = (0.,5.)
                                                                                                                                                           ALPHA=PI/2.
                                                                                                                                                                                                                                                                                                                                                                                                                                                (IHd) NI 5=ES
                                                                                                                                                                                                                                                                                                                                                                                                                                                                  CS=COS (PHI)
                                                                                                         X=RACIUS
                                                                                                                                                                                                                                                                                                                                         (エド) ロスース
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ELTFAC=SIN(1.6*FI*SIN(8ETA)/2%)/(1.6*PI*SIN(8E7A)/2.)
                                                                                                                                                                                                                       APHASE (M) = ATANZ (AIMAG (A (M)), REAL (A (M))) *RAD
                                                                                                                                                                             ALPHA=-ATAN2(-X,SQRT((RADIUS**2-X**2)/E))
                                                                                                                                                                                                                                                                                          - FINDS MAX FIELD AND ANGLE
                                                                                                                                                                                                                                    PPINT 700, THETA, A (M), AMAG (M), APHASE (M) PUNCH 807, THETA, A (M), AMAG (M), APHASE (M)
                                                                                                                                      A (M) =A (Y) +CEXP (RANGE) *PHASE*ELTEAC
                                                                                                                                                                  Z1=SGRT((QADIUS**2.-X**2.)*E)
                                                                                                                                                     X=NP(MM+1)*LAMBDA/(2.*PI)
                                                                                                                                                                                                                                                                                                                                                                                                IF (AMA; (J) -D) 401, 402, 401
IF (BETA-PI/2.)209,232,232
                                                                                                                                                                                                                                                                                                                                                          D=AMAX1(D,AMAG(J))
                                                                                                                                                                                                THETA1=ATAN2(X, Z1)
                                                                                                                                                                                                                                                                                                                                                                                                                                          831,0,THÉTA
                                                                                                                                                                                                             ARAG (M) = CABS:(A(M))
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   804,5 I, NP(I)
                                                                                                                                                                                                                                                                                                                                                                                                                            THETA=FLOAT (JJ-1)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  89.4 4 MW A 9 M 4
                                                                                                                                                                                                                                                                                                                                                                                                                                                                       DO 900 I=1 /HMAX
                                                                                   W=(6.,-1.) *DIST
                                                                                                                                                                                                                                                                     THETA-THETA+1.
                                                                                                                                                                                                                                                                                                                                             00 300 J=2,91
                                                                                                                                                                                                                                                                                                                                                                         DO 421 J=1,91
                                                                                                 Z=((0,10)*PSI
                                                                                                                             PHASE=CEXP(Z)
                                                                                                                                                                                                                                                                                                 THROUGH 492
                                                                                                                                                                                                                                                                                                                                                                                                                                                          865
                                                                                                                                                                                                                                                                                                                                0= ara 6 (1)
                                                                                                                                                                                                                                                                                                                                                                                                                CONTINUE
                                                                                                                RANGE=K*N
                 EL TFAC=0.
                              50 TO 211
                                           ELTFAC=1.
                                                          GO TO 211
                                                                                                                                                                                                                                                                                                                                                                                                                                            PFINT
                                                                                                                                                                                                                                                                                                                                                                                                                                                          PFINT
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F(X)=SQRT((RADIUS**2.-X**2.+E*X**2.)/(RADIUS**2.-X**2.))
                                            DIMENSION A (2), AMAG (2), NP (508), ARC (509)
                                                                                         DEFINE CONSTANTS TO BE USED IN PROGRAM
PROGRAM OFTIMUM (INFUT, CUTFUT)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           CALCULATE GEOMETRY OF ARRAY
                                                                                                                                                                                                                                                                                                     SUMARC=THETA=THETAO=PSI=0.
             COMPLEX A, 1, 2, RANGE, PHASE
                                                                                                                                                                                                       K=2.*PI*F0E0/(3.*10.**8.)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                             RADIUS=N*LAMADA/(2.*PI)
                                                                                                                                                                                       LAMBDA=3.*10.**8./FRE0
                                                                                                                                                                                                                                                                      INITIALIZE VARIABLES
                                                                                                                           I1=I2=I3=I%=I5=0
                                                                                                                                                                         RAD=360./(2.*PI)
                                                                                                                                          FREQ=2.*10.**9.
                              REAL LAMBOA, K, L
                                                                                                                                                        PI=4.*ATAN (1.0)
                                                                                                                                                                                                                                                                                                                                                                                                                                                             PHIO=PHI1=PI/2.
                                                                                                                                                                                                                                                                                                                                  DO 100 I=1,530
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          R=200.*RADIUS
                                                                                                                                                                                                                                                                                                                                                                                DO 110 I=1,2
                                                                                                                                                                                                                                                                                                                                                                                              A (1) = (0.,0.)
                                                                                                                                                                                                                                                                                                                                                                                                               AMAG(I)=0.
                                                                                                                                                                                                                                                                                                                                                 ARC(I)=0.
                                                                                                                                                                                                                                      RATIO=1.
                                                                                                                                                                                                                                                                                                                                                                  Np (I) =0
                                                                                                                                                                                                                                                                                                                                                                                                                                              NP(1)=N
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          E=F1**2
                                                                                                                                                                                                                        E1=1.1
                                                                                                                                                                                                                                                                                                                    N=128
                                                                                                                                                                                                                                                                                                                                                                                                               110
                                                                                                                                                                                                                                                                                                     190
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X=(2.*RADIUS-SORT(LAMBDA**2.*(1.-E)+4.,*E**2.*RADIUS**2.))/(2.*(1.
                                         OF ARRAY TO TOP
                                                                                                                                                                                                                                                                                                                                                                          IF (TARC-SUMARC-LAMBOA/2.)207,207,208
ALPHA=-ATAN2(-X,SQRT((RADIUS**2-X**2)/E)
                                        CALCULATES ARC LENGTH FROM BASE
                                                                                                                                                                                                                                  IF (NP (MM) -NP (MM-1)) 204,205,205
                                                                                                                                                                                                                                                                                                                                                                                                        X1=X-(LAMBDA/2.) *COS(ALPHA)
                                                                                                                                                                                                                                                                                                                       DO 266 II=1,100
ARC(MM)=ARC(MM)+H*F(Y)
                                                                                                                                                                                                                                                                                                                                                                SUMARC=SUMARC+ARC (MM)
                                                                                                                                                                                                                     IF (MM-1) 204,204,203
                                                                                                                                                                                                                                                 NP (MM) = NP (MM-1)-2
                                                                                            TARC=TARC+H*F(X)
                                                                               DC 261 I=1,1000
                                                                                                                                                                                                       NP (MM) = 2*NP (MM)
H=RADIUS/1000.
                                                                                                                                                                                                                                                                 NUM-NUM+NP (MM)
                                                                                                                                                                                           NP(MM)=X*K/2.
                                                                                                                                                                                                                                                                                            H= (X2-X)/100,
                                                                                                                                                                                                                                                                               X-NP (MM) /K
                                                                                                                                                                              X2=RADIUS
                                                                                                                                                                                                                                                                                                                                                                                                                                                             60 TO 209
                                                                                                                                                                                                                                                                                                         Y=X+H/2.
                                                                                                                                                                                                                                                                                                                                                                                                                                                                          CONTINUE
                                                                  TARC=0.
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         HEAX=MR
                                                                                                                                                                                                                                                                                                                                                                                                                                                MF=MM+1
             X=H/2.
                                                                                                           H+X=X
                                                                                                                                                                                                                                                                                                                                                  Y=Y+H
                                                                                                                        NUM=N
                                                                                                                                      ガエージ
                                                                                                                                                                                                                                                                                                                                                                                                                     X5=X
                                                                                                                                                                                                                                                                                                                                                                                                                                    X=X1
                                                                                                                                                                (E)
                                                                                                                                                                                                                                  203
                                                                                                                                                   202
                                                                                                                                                                                           209
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250 LOOP SUMS FIELD CONTRIBUTION FROM EACH SUBARRAY
                                                                                                                                                                                                                                                                                                                                   DO 240 LOOP CALCULATES FIELD FROM EACH SUB ARRAY
            0,90 DEGREES
              CALCULATE FIELD AT THETA =
                                                                                            THETA=PI/2.
                                                                              THETA=9.
                                                                                                                                                                                                                                                                                                                                                                          PHI=2.*PI*(I-1) /N
                                                                                                                                                                                                                                         DO 250 MM=1,MMAX
S1=SIN(ALPHA)
                                                                                                                                                                                                                                                                                                                                                                                                                 SE=SIN(PHI1-PHI)
                                                                                                                                                                                                                                                                                                                                                                                                                                             SRESIN (PHIC-PHI)
                                                                                                        S7=S4=SIN(THETA)
                                                                                                                                                                                                                                                                                                                                                                                                                                Cé=COS (PHI1-PHI)
                                                                                                                    C7=C4=C0S(THETA)
                                                                                                                                                                                                                                                                                SZ=SIN (THETA1)
                                                                                                                                                                                                                                                                                            C2=COS (THETAL)
                                                                                                                                                                                                                                                                    C1=COS (ALPHA)
                                                                                                                                                                                                                                                                                                                                                               00 240 J=1,N
                                                                  30 290 M=1,2
                                                                                                                                                                                      THETA1=PI/2.
                                                                                                                                                            A (M) = (0.,0.
                                                                                                                                                                        ALPHA=PI/2.
                                                                                                                                                                                                                                                                                                                                                                                         CIHO) NIVERS
                                                                                                                                                                                                                                                                                                                                                                                                      (IHd) S00=20
                                                                              IF(M.EQ.1)
IF(M.EQ.2)
                                                                                                                                  X=RADIUS
                                                                                                                                                                                                                                                                                                           N=NP (MM)
                                                                                                                                               Z1=0.
                                                    02=0
                                         S5=1.
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C8=COS (PHIC-PHI)

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E1. IF NOT OPTIMUM STER E1 AND G0 TO 190
          DIST=SQRT((R*S4*C5-X*C3)**2+(R*S4*S5-X*S3)**2+(R*C4-Z1)**2)
BFTA=ACOS((S1*(R*S4*C8-X)+C1*(R*C4-Z1))/DIST)
                                                                                                                                                                ELTFAC=SIN(1.6*PI*SIN(BETA)/2.)/(1.6*PI*SIN(BETA)/2.)
                                                                                                                                                                                                                                                                                                      ALPHA=-ATAN2(-X,SGRT((RADIUS**2-X**2)/E))
                                                                                                                                                                                                                                                       A (M) = A (M) + CEXP (RANGE) * PHASE* ELTFAC
PSI=-K*(21*C7-RABIUS*S7+X*S3*S7)
                                                                                                                                                                                                                                                                                         Z1=SQRT ((RADIUS**2.-X**2.)*E)
                                                                                                                                                                                                                                                                                                                                                                                              TEST FOR OFTEMUM VALUE OF
                                                             IF(BETA)231,233,231
IF(BETA-PI/2,)234,232,232
                                                                                                                                                                                                                                                                                                                                                                                                                                                ÍF (RATIO-AM) 301,302,302
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       RATIO=AMAG (2)./AMAG (1)
                                                                                                                                                                                                                                                                                                                           THETA1=ATAN2 (X, 21)
                                                                                                                                                                                                                                                                                                                                           AMAG (M) =CAES (A (M))
                                                                                                                                                                                                                                                                                                                                                                                                                                  AM=AMAG(2)/AMAG(1)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   řF (11) 304; 304, 305
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         IF(I4)306,306,3117
                                                                                                                                                                                     W=(0.,-1.) *DIST
                                                                                                                                                                                                      Z=(0.,1.)*PSI
                                                                                                                                                                                                                                        PHAŜĒ=CEXP (Z)
                                                                                                                                                                                                                                                                         X=NP(MM+1)/K
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         E1=E1-0.01
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       GO TO 191
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      E1=E1+0.1
                                                                                                                                                                                                                        RANGE=K*N
                                                                                                                   60 TO 211
                                                                                                 ÉLTFAC=0.
                                                                                                                                   ELTFAC=1.
                                                                                                                                                    60 TO 211
                                                                                                                                                                                                                                                                                                                                                            CONTINUE
                                                                                                                                                                                                                                                                                                                                                                                                                                                                     I2=1
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         335
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      304
                                                                                                                                                                                                                                                                                                                                                               290
                                                                                                                                                                                                                                                                                                                                                                                                                                                                     331
                                                                                 231, 232
                                                                                                                                  233
                                                                                                                                                                     234
                                                                                                                                                                                      211
                                                                                                                                                                                                                                                          240
                                                                                                                                                                                                                                                                                                                             250
                                                                                                                                                                                                                                                                                                                                                                                              5000
0000
                                                                                                                                                                                                                                                                                                                                                                              ၁၁၁၁
```

RATIO=AMAG (2) /AMAG (2)

```
W CIRCLE N
                                                                                                                                                                                                                                                                                                                                                             *, F6.4, * METERS*/*
                                                                                                                                                                                                                                                                                                                                                 ARRAY
                                                                                                                                                                                                                                                                                                                                                                                                       *##### NUMBER OF ELEMENTS IN ARRAY ######*
                                                                                                                                                                                                                                                                                                                                                   F
L
                                                                                                                                                                                                                                                                                                                                                               C8.4,* METERS*/* RADIUS OF BASE OF
                                                                                                                                                                                                                                                                                                                                                                                                                      OF ELEMENTS*.)
                                                                                                                                                                                                                                                                                                                                                    FORMAT (1H ,*OFTINUN VALUE OF E1
                                                                                                                                                                                                                                                                                                                                       PRINT 700, E1, HEIGHT, RADIUS, NUN
                                                                                                                                                                                                                                                                                                                                                                             CL NUMBER OF ELEMENTS IN ARRAY
                                                                                                                                                                                                                                                                                         OF ARRAY
                                      RATIO=AMAG (2) /AMAG(1)
                                                                                                                                                                                                                                    RATIC=AMAG (2) /AMAG (1)
                                                                                                                                                                Rat 10= 4 mag (2) / Amag (1)
                                                                                            RAT IO=AMAS (2) /AMAG (1)
                                                                                                                                                                                                                                                                                                                                                                                                                        CUMBER*, 4X, *NUMPER
                                                                IF(12)308,308,309
                                                                                                                                                                                           IF(I5)312,312,313
                                                                                                                       IF(I3)310,310,311
                                                                                                                                                                                                                                                                                                                         HEIGHT = E1* RADIUS
                                                                                                                                                                                                                                                                                         PRINT GEOMETRY
                                                                                                                                                                                                                      E1=E1+0.001
           E1=E1+0.001
                                                                                                                                   E1=E1-0.01
                                                    GO TO 130
                                                                                                                                                                             60 TO 199
                                                                                                                                                                                                                                                 GO TO 190
                                                                                                                                                                                                                                                                                                                                                                                              PRINT 855
                                                                                                                                                                                                                                                                                                                                                                                                          FORMAT (1H
                                                                              E1=E1+0.1
                                                                                                          GO TO 190
GO TO 190
                                                                                                                                                                                                                                                               CONTINUE
                                                                                                                                                                                                         14=1
                                                                                                                                                  I1=1
                           15=1
                                                                                                                                                                                                                                                                                                                                                                                                            805
                                                                                                                       309
                                                                                                                                                                                           311
                                                                                                                                                                                                                                                               313
                                                                  302
308
            307
                                                                                                                                                                                                                                                                             0000
0000
0000
```

FORMAT(1H , 15,9X,15)

(1) aN 6

PPINT 711,1

006

701

Appendix C Geometry of Array

^	
Circle Number	Number of Elements.
1	128
2	126
3	124
4	122
5	120
6	118
7	116
8	114
9	112
10	110
11	108
12	106
13	104
14	102
15	100
16	98
17	96
18	94
19	92
20	90'
21	88
22	. 86
23	84
24	82
25	80

Circle Number	Number of Elements
26	78
27	76
28	74
29	72
30	70
31	68
32	66
33	64
34	62
35	60
36	58
37	56
38	54
39	50
40	46
41	.· 42
42	38
43	34
⁴ 44	30
45	26
46	22
47	18
48	14
49	10
50	6
51	2
f Array 7.6 meters	

Height of Array --- 7.6 meters
Radius of Base of Array --- 3.06 meters
Total Number of Elements in Array --- 3796

X.

The state of the s

Vita

Thomas Burke Markham was born on 1 March 1941 in Memphis,
Tennessee. He graduated from Clarksville High School in Clarksville,
Tennessee, in 1958. He attended Austin Peay State University for two years
prior to enlisting in the United States Air Force in 1960. After completing
technical school he was assigned to Vandenberg Air Force Base, California,
from 1961 to 1965 as a Missile Instrumentation and Range Safety technician
with the Strategic Air Command's 4392nd Communications Squadron. In 1965
he entered Arizona State University under the Airmen's Education and Commissioning Program from which he received a Bachelor of Science in Engineering degree. Upon completion of Officer Training School he was commissioned
a Second Lieutenant in the Air Force Reserve. He served as a Satellite Operations Control Officer from 1967 to 1970. He attended the Air Force Institute
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